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DIVERGENCES OF INTEGRALS FOR GREEN FUNCTIONS OF KLEIN-GORDON AND DIRAC EQUATIONS AND NECESSARY EXISTENCE OF PARTICLE GENERATIONS

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It is shown that the values of the infinite integrals for the Green functions of the Klein – Gordon and Dirac equations depend on the method used for its calculations, i.e., these integrals diverge. The Green functions proposed to eliminate these divergences include in the denominators the polynomials of the N degree instead of one factor $m^2 - q^2$ or $m - \hat{q}$ for the Klein – Gordon equation or the Dirac equation, respectively. The corresponding generalizations of the Klein – Gordon and Dirac equations have $2N$ and N degree, respectively. The solutions of these generalizations for the Klein – Gordon and Dirac equations may be presented by the sum of N terms, each term corresponds to the contribution of one particle (one generation). All these particles have different masses but the same spin, parity, charge, isospin. Since the space – time is 4 – dimensional one, the convergence of the integrals for proposed Green functions is possible only if the generation number N is not less than three for integer spin particles and not less than five for half – integer spin particles. It is shown that the proposed Green functions have no any singularities in the space – time. In particular, the interaction potentials must have the oscillator form at short distance. It is predicted that two (or greater) massive particles with quantum numbers of the photon and gluons must exist. Besides, five (or greater) fermions with quantum numbers of the electron, the neutrino, the $u-$quark, and $d-$quark must exist (e.g., $e_1 = e, e_2 = \mu, e_3 = \tau, e_4, e_5, ...,$ and $u_1 = u, u_2 = c, u_3 = t, u_4, u_5, ...$). The massless neutrino must be one. If higher neutrinos are enough heavy then the decay $\nu_{4,5} \to e\mu\nu_{1,2}$ may be possible.

KEY WORDS: convergence, multiple integrals, Green functions, partial differential equations, oscillatory potentials, particle generations, massive photon, massive gluons, massive neutrino.

РАСХОДИМОСТИ ИНТЕГРАЛОВ ДЛЯ ФУНКЦИЙ ГРИНА УРАВНЕНИЙ КЛЕЙНА – ГОРДОНА И ДИРАКА И НЕОБХОДИМОСТЬ СУЩЕСТВОВАНИЯ ПОКОЛЕНИЙ ЧАСТИЦ

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Показано, что значения несобственных интегралов для функций Грина уравнений Клейна – Гордона и Дирака зависят от метода вычисления, т.е. эти интегралы расходятся. Функции Грина предложенные для устранения расходимостей содержат в знаменателях полиномы степени N вместо одного множителя $m^2 - q^2$ или $m - \hat{q}$ для уравнений Клейна – Гордона и Дирака, соответственно. Соответствующие обобщения уравнений Клейна – Гордона и Дирака имеют порядки $2N$ и N, соответственно. Решения этих обобщений уравнений Клейна – Гордона и Дирака можно представить в виде суммы N слагаемых, каждый из которых соответствует вкладу одной частицы (одного поколения). Все эти частицы имеют одни и те же значения спина , четности, заряда изоспина, но разные массы. Так как пространство-время имеет четыре измерения, то сходимость интегралов для предложенных функций Грина возможна только если количество поколений N не меньше трех для частиц с целым спином и не менее пяти для частиц с полуцелым спином. Показано, что предложенные функции Грина не имеют сингулярностей во всем пространстве – времени. В частности, потенциалы взаимодействий должны иметь осцилляторный вид на малых расстояниях. Предсказывается, что должны существовать две (или больше) массивных частиц с квантовыми числами фотона и глюона. Кроме того, должны существовать пять (или больше) фермионов с квантовыми числами электрона, нейтрино, u - кварков и d - кварков например, $e_1 = e, e_2 = \mu, e_3 = \tau, e_4, e_5, \ldots$ и $u_1 = u,\ u_2 = c,\ u_3 = t, u_4, u_5, \ldots$). Безмассовое нейтрино может быть только одно. Если высшие нейтрино имеют достаточно большие массы, то возможен распад $\nu_{4,5} \to e\mu\nu_{1,2}$.

КЛЮЧЕВЫЕ СЛОВА: сходимость, кратные интегралы, функции Грина, дифференциальные уравнения в частных производных, осциллятоорные потенциалы, массивный фотон, массивный глюон, массивное нейтрино.

РОЗБІЖНОСТІ ІНТЕГРАЛІВ ДЛЯ ФУНКЦІЙ ГРІНА РІВНЯНЬ КЛЕЙНА - ГОРДОНА ТА ДІРАКА І НЕОБХІДНІСТЬ ІСНУВАННЯ ПОКОЛІНЬ ЧАСТИНОК

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Показано, що значення невласних інтегралів для функцій Гріна рівнянь Клейна - Гордона та Дірака залежать від методу обчислення, тобто ці інтеграли розбігаються. Функції Гріна запропоновані для усунення розбіжностей містять у знаменниках поліноми ступеня N замість одного множника m^2-q^2 або $m-\hat{q}$ для рівнянь Клейна - Гордона та Дірака, відповідно. Відповідні узагальнення рівнянь Клейна – Гордона і Дірака мають порядки $2N$ та N, відповідно. Розв'язки цих узагальнень рівнянь Клейна - Гордона і Дірака можна представити у вигляді суми N доданків, кожен з яких відповідає внеску однієї частинки (одного покоління). Всі ці частинки мають одні й ті ж значення спіну , парності, заряду, ізоспіну, але різні маси. Оскільки простір-час має чотири виміри, то збіжність інтегралів для запропонованих функцій Гріна можлива тільки якщо кількість поколінь N не менша трьох для частинок із цілим спіном та не менше п'яти для частинок із пів-цілим спіном. Показано, що запропоновані функції Гріна не мають сингулярностей у всьому просторі - часу. Зокрема, потенціали взаємодій повинні мати осцилляторний вигляд на малих відстанях. Передбачено, що дві (або більше) масивних частинок з квантовими числами фотона і глюона повинні існувати. Окрім цього п'ять (або більше) ферміонів з квантовими числами електрона, нейтрино, u - кварків та d - кварків повинні існувати (наприклад, $e_1=e,\ e_2=\mu,\ e_3=\tau,\ e_4,\ e_5,\ldots$ та $u_1=u,\ u_2=c,\ u_3=t,\ u_4,\ u_5,\ldots$). Безмасове нейтрино може бути тільки одне. Якщо вищі нейтрино мають досить великі маси, то можливий розпад $\nu_{4,5}\to e\mu\nu_{1,2}$.

КЛЮЧОВІ СЛОВА: збіжність, кратні інтеграли, функції Гріна, диференціальні рівняння в частинних похідних, осциляторні потенціали, масивний фотон, масивний глюон, масивне нейтрино.

In the quark model it has been shown that the hadrons consist of the quarks of six flavors. Therefore, now only the leptons, the quarks, the photon, the gluons, $W^{\pm}$, and Z^0 are considered as elementary particles. The study of the axial Adler – Bell – Jackiw anomaly shown that the contribution of one $1/2$ - spin particle (a quark or a lepton) gives a linear divergence [1]. But taking into account of some sets of leptons and quark, such as e, ν_e, u, d or μ, ν_μ, c, s or τ, ν_τ, t, b, allows to eliminate this divergency. Thus, the convergence of the axial anomaly gives the relation between the quarks and the leptons.

In connection with this the question arises: why do the generations of particles exist? We can remember the words of L.B. Okun that we good understand the reasons for the existence of some new particles. But we do not understand: why do old particles (for example, the muon) exist. Now the generations of the fermions are known only. Therefore, the question is arisen: do the generations of bosons exist or not? In Ref. [2] the results of the investigations for the decays of heavy neutral particle into $\mu^+\mu^-X$ are presented. These results may be considered as the manifestation of now neutral boson $Z^{0\,\prime}$, which participates in the weak interactions. The existence of such weak-interacting boson may be great of importance, as then it is clear that the Standard Model is not complete. In relation with this it is of interest to investigate the theoretical reasons for the existence of the fermion and boson generations.

At present paper the bases of the elementary particles theory are studied to answer on the question related to the existence of the particle generations. We show that the particle generations must exist.

PARADOX OF THE GREEN FUNCTIONS

Let us consider the particle propagators, i.e., the Green functions. It is well known that in the static case the exchange by the massless particle gives the Coulomb (Newton) potential

$$V(r,0)=\frac{1}{4\pi r}, \tag{1}$$

where r is the distance between the point charge and the point of observation.

The exchange by the particle of the mass m gives the Yukawa potential

$$V(r,m)=\frac{1}{4\pi}\frac{e^{-mr}}{r}. \tag{2}$$

These potentials are the Green functions

$$V(r,m)=G(\vec{x},m)=\frac{1}{(2\pi)^3}\int\frac{e^{i\vec{q}\vec{x}}}{\vec{q}^{\,2}+m^2}d^3q, \tag{3}$$

where $r=|\vec{x}|$. Note that we can put $m=0$ in Eqs. (2), (3) for the Coulomb potential. In the relativistic case the exchange by the boson of the mass m can be expessed by means of the Green function for the Clein-Gordon-Fock equation

$$D(x,m)=\frac{1}{(2\pi)^4}\int\frac{e^{-iqx}d^4q}{-q^2+m^2}\quad. \tag{4}$$

For the $1/2$ - spin particle the Green function of the Dirac equation has a form

$$S(x,m)=\frac{1}{(2\pi)^4}\int\frac{\left(\hat{q}+m\right)e^{-iqx}d^4q}{-q^2+m^2}. \tag{5}$$

Usually the expressions (1) and (2) are derived from Eq. (3) by the calculations of the integrals in the spherical frame. Note that the integral in Eq. (3) is the infinite threefold integral. As it is known, the improper (in particular infinite) integral converges in that case only if the calculations of it give the same finite result by any possible methods. The convergences of improper onefold and multiple integrals have some distinctions. Thus, for multiple improper integral the conditional convergence does not exist. In Refs.[3, 4] it is proved that if the twofold improper integral converges then it converges also absolutely (i.e., the improper twofold integral with the module of the integrand converges). This is valid for any multiple improper integral too [4]. Thus, for the multiple improper integrals the convergence and the absolute convergence are equivalent [4]. Therefore, multiple improper integral converges then and only then when this integral converges absolutely. Thus, the integral in Eq. (3) converges only in the case of the convergence of the integral

$$\frac{1}{(2\pi)^3}\int\frac{d^3q}{\vec{q}^{\,2}+m^2}. \tag{6}$$

But this integral diverges. Therefore, the integral in Eq. (3) diverges. To see this immediately we integrate Eq. (3) in the cylindrical frame. We choose $\vec{x}=(0,0,r)$. Then $\vec{q}\vec{x}=q_3r$ and $d^3q=\frac{1}{2}d\left|\vec{q}_\perp\right|^2 d\varphi\, dq_3$. We integrate in the next order: with respect to the angle φ, $\left|\vec{q}_\perp\right|$, q_3, respectively. Thus, we derive

$$\begin{aligned}G(\vec{x},m)&=\frac{1}{8\pi^2}\int\limits_{-\infty}^{+\infty}e^{iq_3r}dq_3\int\limits_0^{\infty}\frac{d\vec{q}_\perp^{\,2}}{\vec{q}_\perp^{\,2}+q_3^2+m^2}=\\&=\frac{1}{8\pi^2}\int\limits_{-\infty}^{+\infty}e^{iq_3r}dq_3\left[\lim_{\vec{q}_\perp^{\,2}\to\infty}\ln\left(\vec{q}_\perp^{\,2}+q_3^2+m^2\right)-\ln\left(q_3^2+m^2\right)\right]=\\&=\frac{1}{2\pi}\delta(r)\lim_{\left|\vec{q}_\perp\right|\to\infty}\ln\left|\vec{q}_\perp\right|-\frac{1}{2\pi^2r}\lim_{q_3\to\infty}\ln q_3\sin q_3r+\frac{1}{4\pi}\cdot\frac{e^{-mr}}{r}.\end{aligned} \tag{7}$$

We see that this integral diverges since the first term is indefinite and the limit in the second term does not exist, but these diverging terms do not depend on the particle mass. Note that the infinite three-fold integrals m,s (3), (7) converges only in the case if calculations of it by all possible methods give the same finite value. In particular, the limits in Eq. (7) must have the same value when $q_1\to\pm\infty,\ q_2\to\pm\infty,\ q_3\to\pm\infty$, $\left(\left|\vec{q}_\perp\right|\to\infty\right)$.

For the Green function of the Klein – Gordon equation we calculate the integral (4) with respect to the spatial variables by integration in the cylindric frame similarly to the integral (7). Then we derive

$$\begin{aligned}D(x,m)&=\frac{1}{16\pi^3}\int\limits_{-\infty}^{\infty}dq_0e^{-iq_0x_0}\int\limits_{-\infty}^{\infty}dq_3e^{iq_3x_3}\int\limits_0^{\infty}\frac{d\vec{q}_\perp^{\,2}}{\vec{q}_\perp^{\,2}+q_3^2-q_0^2+m^2}=\\&=\frac{1}{16\pi^3}\int\limits_{-\infty}^{\infty}dq_0e^{-iq_0x_0}\int\limits_{-\infty}^{\infty}dq_3e^{iq_3x_3}\left[\lim_{\vec{q}_\perp^{\,2}\to\infty}\ln\left|\vec{q}_\perp^{\,2}+q_3^2-q_0^2+m^2\right|-\ln\left|q_3^2-q_0^2+m^2\right|\right].\end{aligned} \tag{8}$$

In last improper integral the indefinites related to the transitions $\left|\vec{q}_\perp\right|\to\infty,\ q_3\to\pm\infty,\ q_0\to\pm\infty$ arise as addition to the indefinites in Eq. (7). They are the additional of sources of the divergence for the Green function of the Klein – Gordon equation.

From the comparison of the integrands in Eq. (4) and Eq. (5) we see that the integrand in Eq. (5) includes the additional factor $\hat{q}$ which is related to the integration variables. Therefore, the improper integral (5) for the Green function of the Dirac equation diverges also.

Thus, we derive the paradox (paradox of the Green functions):
From the mathematical point of view the use of the Green functions (1)-(5) is incorrect, but these Green functions (calculated by some fashion) give adequate description of different experimental data.

We may assume that the solution of the Green function paradox is possible by two ways: 1) We can conclude that existing theory is wrong and we must find new theoretical approach based on new mathematical methods; 2) We can try to modify existing theory.

GENERALIZATIONS OF KLEIN – GORDON AND DIRAC EQUATIONS

We consider second way by means of proper modification of the Green functions and corresponding generalization of the Klein-Gordon and Dirac equations. We propose:

1) The generalizations of the Klein-Gordon and Dirac equations must have some simple form;
2) The existing expressions (such as Eqs. (1), (2)) can be derived from new generalized Green functions in some limit.

We propose that the generalized non-homogeneous Klein-Gordon equation has $2N$ degree, and may be written as

$$\left(\square+m_1^2\right)\left(\square+m_2^2\right)........\left(\square+m_N^2\right)\varphi\left(x\right)=\eta\left(x\right), \tag{9}$$

where $\varphi(x)$ is the field and $\eta(x)$ is the current (the field source). The Green function for Eq. (9) is given by

$$\bar{G}\left(x\right)=\frac{1}{\left(2\pi\right)^4}\int\frac{e^{-iqx}d^4q}{\left(-q^2+m_1^2\right)\left(-q^2+m_2^2\right)....\left(-q^2+m_N^2\right)}=\frac{1}{\left(2\pi\right)^4}\int\frac{e^{-iqx}d^4q}{P_N\left(q^2\right)}, \tag{10}$$

where $P_N\left(q^2\right)$ is the polynomial of the degree N with respect to q^2. We consider the case of the polynomial with real non-negative different zeros $m_1<m_2<m_3<....<m_N$. Note that for the advanced, retarded and causal Green functions we must write the corresponding imaginary infinitesimal term to each q_0 or m_k^2, respectively.

The general classical solution $\varphi_{cl}(x)$ of the linear equation (9) is the sum of the general solution of the corresponding homogeneous equation $\varphi(x)_{free}$ and partial solution $\varphi(x)_{nh}$ of non-homogeneous equation:

$$\varphi\left(x\right)_{free}=\int d^4q\sum_{k=1}^{N}\delta\left(q^2-m_k^2\right)\left[c_ke^{-iqx}+\tilde{c}_ke^{iqx}\right], \tag{11}$$

$$\varphi\left(x\right)_{nh}=\int d^4y\bar{G}\left(x-y\right)\eta\left(y\right)d^4y, \tag{12}$$

where c_k and $\tilde{c}_k$ are the arbitrary constants. Thus, $\varphi(x)_{free}$ is the sum of the terms corresponding to particles with the same charges, parity, but with different masses. Each term in Eq. (11) corresponding to the index k is the solution of the homogeneous Klein – Gordon equation: $\left(\square+m_k^2\right)\left(c_ke^{-iqx}+\tilde{c}_ke^{-iqx}\right)\ \delta\left(q^2-m_k^2\right)=0$. We exclude the case of equal masses in Eqs. (9) and (10). We can show that the functions $\varphi(x)_{free}$ are non- normalizable if at least two masses are equal. Consider the equation similar to Eq. (9) in the case of n equal mass m (i.e., m is n - fold root):

$$\left(\square+m^2\right)^n\left(\square+m_2\right)\ldots\left(\square+m_{N-n}\right)\varphi\left(x\right)=\eta\left(x\right). \tag{13}$$

The part of the classical solution $\varphi(x)_{free}$ corresponding to the equal masses may be written in general form similarly to Eq. (11):

$$\begin{aligned}&\varphi\left(x\right)_{n,free}=\int d^4q\delta\left(q^2-m^2\right)\cdot\\&\Big\{\left[C_1+C_2\left(xa_1\right)+C_3\left(xa_{21}\right)\left(xa_{22}\right)+\ldots+C_n\left(xa_{n-1,1}\right)\left(xa_{n-1,2}\right).....\left(xa_{n-1,n-1}\right)\right]e^{iqx}+\\&+\left[\widetilde{C}_1+\widetilde{C}_2\left(xb_1\right)+\widetilde{C}_3\left(xb_{21}\right)\left(xb_{22}\right)+\ldots+\widetilde{C}_n\left(xb_{n-1,1}\right)\left(xb_{n-1,2}\right)....\left(xb_{n-1,n-1}\right)\right]e^{-iqx}\Big\},\end{aligned} \tag{14}$$

where $C_1,C_2,\ldots,C_n,\widetilde{C}_1,\widetilde{C}_2,\ldots,\widetilde{C}_n$ are arbitrary constants, and $a_1,\,b_1,\,a_{21},a_{22},\,b_{21},\,b_{22},...,a_{n-1,1},a_{n-1,2},$ $a_{n-1,n-1},\,b_{n-1,1},b_{n-1,2},b_{n-1,n-1}$ are arbitrary 4 - vectors, which do not depend on the components of the 4 - vector x. As it is known, $\left|\varphi(x)\right|^2$ is the probability density and

$$\iiint\limits_V\left|\varphi\left(x\right)\right|^2d^3x=1, \tag{15}$$

i.e., one particle is in the volume V. For each term in the solution (11) it is easy to derive

$$\frac{1}{V}\iiint\limits_V\left|\exp\left(\pm iqx\right)\right|^2d^3x=1. \tag{16}$$

But for one term in Eq. (14) we have (at equal vectors $a_{k1},a_{k2},...,a_{kk}$):

$$\iiint\limits_V\left|\left(xa\right)^k\exp\left(\pm iqx\right)\right|^2d^3x,\ k=1,2,\ldots,n-1. \tag{17}$$

These integrals depend on the time x_0 and we cannot find some constant to derive 1 for such integrals. Moreover,

$$\lim_{V\to\infty}\iiint\limits_V\left|\left(xa\right)^k\exp\left(\pm iqx\right)\right|^2d^3x=\infty \tag{18}$$

The non-normalizable solutions of Eq. (13) can be eliminated in the case of non-zero constants C_1l and $\widetilde{C}_1$ only in the solution (14). But in this case we derive the solutions of Eq. (13) at $n=1$. Then we may introduce new current $\widetilde{\eta}\left(x\right)=\left(\square+m^2\right)^{n-1}\eta\left(x\right)$. Thus for the normalizable solutions the Eq. (13) is reduced to the Eq. (9). Therefore, for the solution in case of equal masses the relation (15) cannot be valid. Thus, the masses in the generalized Klein Gordon equation must be different. We can write

$$\frac{1}{P_N\left(q^2\right)}=\frac{1}{\left(-q^2+m_1^2\right)\left(-q^2+m_2^2\right)...\left(-q^2+m_N^2\right)}=\sum_{k=1}^{N}\frac{A_k}{-q^2+m_k^2},$$
$$A_k=-\frac{1}{P_N'\left(m_k^2\right)}=\lim_{q^2\to m_k^2}\frac{-q^2+m_k^2}{P_N\left(q^2\right)},\quad A_k=\left(-1\right)^{k+1}\left|A_k\right|. \tag{19}$$

For the coefficients A_k the following relations are valid:

$$\sum_{k=1}^{N}A_k m_k^{2l}=0,\qquad l=0,1,2,...,N-2, \tag{20}$$

$$\sum_{k=1}^{N}A_k m_k^{2N-2}=1\,. \tag{21}$$

To prove the relations (20), (21) we introduce $z=1/q^2$ and use the expantion of the fractions in the geometrical series. Then we can expand Eq. (19) in the power series

$$\frac{z^N}{\left(1-m_1^2z\right)\left(1-m_2^2z\right)\cdots\left(1-m_N^2z\right)}=\sum_{n=1}^{\infty}a_nz^n, \tag{22}$$

and

$$a_n=(-1)^{N+1}\sum_{k=1}^{N}A_k m_k^{2n-2}\,. \tag{23}$$

We see that the series in the left side of Eq. (22) begin from $n=N$, but in the right side begin from $n=1$. Therefore, the coefficients $a_n=0$ for $n=1,2,\ldots,N-1$. It gives the relations (20). In addition we have $a_N=1$, i.e., the relation (21) is valid. Using the equality (19) we can express the Green function (10) of Eq. (8) in terms of the Green functions (4)

$$\overline{G}\left(x\right)=\sum_{k=1}^{N}A_kG\left(x,m_k\right). \tag{24}$$

As the dimension of the time-space is equal to four the integral (10) can be convergent at $N\geq 3$. Consequently for each spinless particle two (or greater) particles with the same charges, isospin, C - and P parity, but different masses, must exist in addition. We may say that such particles are members of some set (a family or a kind or a dynasty). The members of different kinds belong to the generation. In Eqs. (11), (19) k is the number of the particle generation. We may assume that the numbers of the members for the elementary particle kinds are less than the member number for the composite particle kinds. Each particle belongs to some kind and some generation.

For the $\frac{1}{2}$ - spin particles we propose the next genertization of the non-homogeneous Dirac equation

$$\left(-i\hat{\partial}+m_1\right)\left(-i\hat{\partial}+m_2\right).....\left(-i\hat{\partial}+m_N\right)\psi\left(x\right)^{\alpha}=\chi\left(x\right)^{\alpha}, \tag{25}$$

where α is the bispinor index.
The classical solution of the homogeneous equation (25) is given by analogy with Eq. (11)

$$\psi\left(x\right)_{free}=\sum_{s}\sum_{k=1}^{N}\int d^4p\delta\left(q^2-m_k^2\right)\left[C_{k,s}u_{k,s}\left(q\right)e^{-iqx}+\widetilde{C}_{k,s}v_{k,s}\left(q\right)e^{iqx}\right], \tag{26}$$

where s corresponds to the spin projection, $u_{k,s}\left(q\right)$ and $v_{k,s}\left(q\right)$ are the spinors, $C_{k,s}$ and $\widetilde{C}_{k,s}$ are arbitrary constants. The Green function for this equation may be written as

$$\overline{S}\left(x\right)=\frac{1}{\left(2\pi\right)^4}\int\frac{\left(\hat{q}+m_1\right)\left(\hat{q}+m_2\right)....\left(\left(\hat{q}+m_N\right)\right)}{\left(-q^2+m_1^2\right)\left(-q^2+m_2^2\right)...\left(-q^2+m_N^2\right)}d^4q\,. \tag{27}$$

For the integrand in Eq. (27) we may write

$$R_N\left(\hat{q}\right)=\frac{\left(\hat{q}+m_1\right)}{\left(-q^2+m_1^2\right)}\cdot\frac{\left(\hat{q}+m_2\right)}{\left(-q^2+m_2^2\right)}\cdots\frac{\left(\hat{q}+m_N\right)}{\left(-q^2+m_N^2\right)}=\frac{1}{\left(-\hat{q}+m_1\right)\left(-\hat{q}+m_2\right)\cdots\left(-\hat{q}+m_N\right)}=\frac{1}{Q_N\left(\hat{q}\right)}=$$
$$=\sum_{k=1}^{N}B_k\frac{\hat{q}+m_k}{-q^2+m_k^2},\qquad B_k=-\frac{1}{Q_N^{/}\left(m_k\right)}=\lim_{\hat{q}\to m_k}\frac{-\hat{q}+m_k}{Q_N\left(\hat{q}\right)},\quad B_k=\left(-1\right)^{k+1}\left|B_k\right|. \tag{28}$$

For the B_k coefficients the relations similar to Eqs. (20), (21) are valid:

$$\sum_{k=1}^{N} B_k m_k^l = 0, \text{ for } l = 0,1,2,\ldots,N-2, \tag{29}$$

$$\sum_{k=1}^{N} B_k m_k^{N-1} = 1. \tag{30}$$

For the Green function (27) we may write

$$\overline{S}(x) = \sum_{k=1}^{N} B_k S(x, m_k) = \sum_{k=1}^{N} B_k \left(i\hat{\partial} + m_k\right) D(x, m_k). \tag{31}$$

The integral (27) can be convergent at $N \geq 5$ only. Thus, for each spin - $\frac{1}{2}$ particle four (or greater) particles with different masses but with the same charges, spin, P - parity must exist in addition.

ABSENCE OF SINGULARITIES IN GREEN FUNCTIONS OF GENERALIZED KLEIN – GORDON AND DIRAC EQUATIONS

Since the generalized Klein-Gordon equation (9) and generalized Dirac equation (19) have degree greater than four their Green functions and their first partial derivatives can be continuous function of the time and spatial variables, i.e., these Green functions cannot have any singularities (more precisely these Green functions can have the removable discontinuity). Note that the Green functions of the Klein-Gordon equation have singularities on the light cone, such as $\delta\left(x^2\right)$, $1/x^2$, $\Theta\left(x^2\right)$, $\ln\left|x^2\right|$ [5, 6]. The singularities disappear in causal $\overline{D}(x)_c$, advanced $\overline{D}(x)_{adv}$, and retarted $\overline{D}(x)_{ret}$ by similar fashion. For example, using the expression for the causal Green function of the Klein – Gordon equation [5] near light cone and Eq. (24) we derive

$$\begin{gathered}\overline{G}(x)^c = \sum_{k=1}^{N} A_k \left[\frac{1}{4\pi}\delta\left(x^2\right) + \frac{1}{4\pi^2 i}\frac{1}{x^2} - \frac{m_k^2}{16\pi}\Theta\left(x^2\right) + \right. \\ \left. +i\frac{m_k^2}{8\pi^2}\left[\ln m_k + \ln\left(\sqrt{\left|x^2\right|}/2\right)\right]\right] = \frac{i}{8\pi^2}\sum_{k=1}^{N} A_k m_k^2 \ln m_k .\end{gathered} \tag{32}$$

The singular terms including $\delta\left(x^2\right)$, $\frac{1}{x^2}$ (and $\Theta\left(x^2\right), \ln\left|x^2\right|$) disappear in $\overline{G}(x)^c$ as consequence of the relation (20) at $l = 0$ (and $l = 1$). The continuous casual Green function may be written as

$$\overline{G}(x)^c_{cont} = \begin{cases} \dfrac{i}{8\pi^2}\displaystyle\sum_{k=1}^{N} A_k m_k^2 \ln m_k, & x^2 = 0, \\ \displaystyle\sum_{k=1}^{N} A_k G_c\left(x, m_k\right), & x^2 \neq 0. \end{cases} \tag{33}$$

Similarly, the elimination of singularities in the Green function of generalized Dirac equation can be derived. In particular, for continuous causal Green function we have in accordance with Eqs. (31) and (32):

$$\overline{S}(x)^c_{cont} = \begin{cases} \displaystyle\sum_{k=1}^{N} B_k m_k^3 \ln m_k, & x^2 = 0 \\ \displaystyle\sum_{k=1}^{N} B_k S\left(x, m_k\right)^c, & x^2 \neq 0 \end{cases} \tag{34}$$

The elimination of the singularities for the Green function can be shown in the static case too. Similarly to Eqs. (9), (10), (24) we have the generalization of the Yukawa potential

$$\overline{G}\left(\vec{x}\right) = \sum_{k=1}^{N} A_k G\left(\vec{x}, m_k\right) = \frac{1}{4\pi}\sum_{k=1}^{N} A_k \frac{e^{-m_k r}}{r}. \tag{35}$$

Each term of the sum in Eq. (35) has singularity at $r = \left|\vec{x}\right| = 0$ (i.e., on the light cone $x^2 = 0 - r^2 = 0$). Using the expansion $e^{-m_k r} = 1 - m_k r + \frac{m_k^2 r^2}{2} - \frac{m_k^3 r^3}{6} + \ldots$ at small r and relations (20) for $l = 0$ and 1 we derive

$$\overline{G}\left(\vec{x}\right) \approx -\frac{1}{4\pi}\sum_{k=1}^{N} A_k \left(m_k + \frac{m_k^3 r^2}{6}\right). \tag{36}$$

Note that the change of the sigh in A_k corresponds to mutual change of the attraction and the repulsion in the interaction potential (35) with the number k. The continuous $\overline{G}\left(\vec{x}\right)$ is given by

$$\overline{G}\left(\vec{x}\right)_{cont}=\begin{cases}-\dfrac{1}{4\pi}\sum_{k=1}^{N}A_k m_k, & r=0,\\ \dfrac{1}{4\pi}\sum_{k=1}^{N}A_k\dfrac{e^{-m_k r}}{r}, & r\neq 0.\end{cases}\tag{37}$$

The function $\overline{G}\left(\vec{x}\right)$ has no any singularities, as contrast with the Goulomb and Yukawa potentials. From Eq. (36) we see that the potential must have the oscillatory form at short distances. Note that, the oscillatory potentials are widely used in the nuclear physics and in the quark models [7, 8]. But in these cases the parameters of the oscillatory potentials are determinated from the experimental data. The interaction force at small r

$$\vec{F}\left(\vec{x}\right)=-grad\,\overline{G}\left(\vec{x}\right)=\frac{\vec{x}}{12\pi}\sum_{k=1}^{N}A_k m_k^3\tag{38}$$

has no any singularities too. It is interesting to note that $\vec{F}\left(0\right)=0$. Thus, we see that the exchange by all the particles from boson kind leads to the relaxation of the interaction at short distances. This is similar to the asymptotic freedom.

Note that if to use for $V\left(r,m_k\right)$ the result (7) derived in the cylindrical frame instead of Eq. (2) then the contributions of the diverging terms vanish, as consequence of the relation (20) at $l=0$. Thus we derive Eqs. (35), (36) using for $V\left(r,m_k\right)$ Eq. (2) and Eq. (7). This confirms the convergence of $\overline{G}\left(\vec{x}\right)$.

LOW – ENERGY LIMITS

Consider the question about the reproduction of the results derived early (such as Eqs. (1) and (2)) in our approach. It is easy to see from Eq. (35) that at relatively large r in the sum the term including m_1 is important only, i.e., at relatively large r $\overline{G}(\vec{x})$ approximately is equal to the Yukaua potential. Simultaneously large r corresponds to small components of the q - momentum. Assume that $m_1/m_k\ll 1$ for $k=2,3,...,N$. Then we can rewrite approximately the equations (9) and (25) in forms

$$\begin{aligned}&\left(\Box+m_1^2\right)m_2^2...m_N^2\varphi\left(x\right)=\eta\left(x\right),\\&\left(-i\hat{\partial}+m_1\right)m_2...m_N\psi\left(x\right)=\chi\left(x\right).\end{aligned}\tag{39}$$

These equations practically coincide with the non-homogeneous Klein-Gordon and Dirac equations for the particles with the mass m_1.

We can reduce at large distances (i.e., in low energy approximation) the equations (9) and (25) to the non-homogeneous Klein –Gordon and Dirac equations, respectively, by means of the redefinitions of the interaction currents. We have seen from Eq. (7) that the calculations of the Coulomb and Yukawa potentials (1), (2) by means of the integral (3) are incorrect. But we derived these potentials as large-distance limit of the Green function for the generalization of the Klein – Gordon equation in the static case (35). In consequence of this and approximate validity of the Klein –Gordon equation at low energies (at large distances), we may assume that the use of the Coulomb and Yukawa potentials in the low energy physics is admissible. In particular, the results derived in the solid state physics, the plasma physics, the statistical physics, the atomic physics, and low energy nuclear physics are valid.

EQUATIONS FOR KINDS OF BOSOS WITH HIGHER SPIN

The interactions of the higher-spin particles usually described by the systems of non-homogeneous Klein-Gordon and Dirac equations. We consider the higher spin particles with integer spin $J=l$ $\left(J\geq 1\right)$ in the Rarita-Schwinger formalism. The fields corresponding to these particles are described by the tensors $U\left(x\right)_{\mu_1...\mu_l}$ and $U\left(q\right)_{\mu_1...\mu_l}$ (in the coordinate and the momentum representations, respectively). These tensors are symmetric, traceless and

$$\partial_{\mu_i}U\left(x\right)_{\mu_1...\mu_l}=0,\qquad q_{\mu_i}U\left(q\right)_{\mu_1...\mu_l}=0,\tag{40}$$

where $i=1,2,...,l$.

The generalization of the Klein-Gordon equation system for the kind of the particles with arbitrary integer spin and masses may be written similarly to Eq. (9) in agreement with Refs. [9, 11]

$$\left(-\Box\right)^l\Pi\left(x\right)_{\mu_1...\mu_l,\nu_1...\nu_l}\left(\Box+m_1^2\right)\left(\Box+m_2^2\right)...\left(\Box+m_N^2\right)U\left(x\right)_{\mu_1...\mu_l}=\tilde{j}\left(x\right)_{\mu_1...\mu_l},\tag{41}$$

where $\Pi\left(x\right)_{\mu_1...\mu_l,\nu_1...\nu_l}$ is the projection operator. Eqs. (9), (25), (41) are written for all the particles from some kind. In Refs. [9-11] it is shown that the physical currents $\tilde{j}\left(x\right)_{\mu_1...\mu_l}$ must obey the theorem on currents and fields as well as the theorem on current asymptotics. Accordingly to the theorem on currents and fields the tensors of the physical currents must have the same properties as the field tensors:

$$\partial_{\mu_i} \widetilde{j}\left(x\right)_{\mu_1 \dots \mu_l} = 0, \qquad q_{\mu_i} \widetilde{j}\left(q\right)_{\mu_1 \dots \mu_l} = 0, \tag{42}$$

$$g_{\mu_i \mu_k} \widetilde{j}\left(x\right)_{\mu_1 \dots \mu_l} = 0, \qquad g_{\mu_i \mu_k} \widetilde{j}\left(q\right)_{\mu_1 \dots \mu_l} = 0. \tag{43}$$

The coordinate representation — The momentum representation

We consider the higher spin particles which move along the z -axis (i.e., $q = \left(q_0, 0, 0, q_3\right)$). Then the integrals

$$\int_{-\infty}^{\infty} dq_0 \int_{-\infty}^{\infty} \left|\widetilde{j}\left(q\right)_{\mu_1 \dots \mu_l}\right| \left|q_0\right|^{\overline{m}_0} \left|q_3\right|^{\overline{m}_3} dq_3 \tag{44}$$

must be convergent in accordance with Ref. [9]. The convergence of the integrals Eq. (44) correspond to theorem on current asymptotics. In the integrals Eq. (44) $\overline{m}_0$ and $\overline{m}_3$ are the integer non-negative numbers, $\overline{m}_0 + \overline{m}_3 = 0, 1, \dots, \overline{m}\left(j\right)$. In Ref. [9] it is derived that $\overline{m}\left(j\right) = 1$. Now we demand that the double Fourier transformation of the current components must converge to the values of these components in any space-time point.

But it is possible if these components of $\widehat{j}\left(x\right)_{\mu_1 \dots \mu_l}$ are continuous and have continuous derivatives $\frac{\partial}{\partial x_0} \widetilde{j}\left(x\right)_{\mu_1 \dots \mu_l}$, $\frac{\partial}{\partial x_3} \widetilde{j}\left(x\right)_{\mu_1 \dots \mu_l}$, $\frac{\partial^2}{\partial x_0 \partial x_3} \widetilde{j}\left(x\right)_{\mu_1 \dots \mu_l}$ [3]. Therefore, we may derive that $\overline{m}\left(j\right) = 3$. The convergence of the integrals Eq (40) is provided by form factors. The currents for the interaction of higher spin bosons with two spinless particles are derived in Refs. [12, 13]. These currents obey the theorem on currents and fields as well as the theorem on current asymptotic. This model allows to derive the finite value of the self-energy operator of the spinless particle corresponding to the contributions of the higher spin boson and spinless particle in one-loop approximation [11]. Note that, as it is well known, the contribution of two spinless virtual particles to this operator in one-loop approximation gives logarithmic divergence. From Eq. (41) we see that the maximal degree of the derivatives of the physical currents is the same as the similar degree for the physical currents in Refs.[9-11]. Therefore, the number $\overline{m}\left(j\right)$ does not change.

Now we consider the equation system for the kinds of the 1- and 2-spin particles, which include the massless particles (i.e., $m_1 = 0,\ 0 < m_2 < m_3 < \dots < m_N$). To derive the equation system we may put $m_1 = 0$ in Eq. (41). But using the properties of the $U\left(x\right)_\mu$ and $U\left(x\right)_{\mu_1 \mu_2}$ (in particular, Eq. (40)) we derive equations inluging the operators $\Box^2$ and $\Box^3$, respectively. Thus, we obtain the equations similar to Eq. (13) with the mass $m = 0$ and the degrees $n = 2,3$, respectively. We regard the constants $C_1 \neq 0$ and $\widetilde{C}_1 \neq 0$ in the solution Eq. (14) only to avoid the non-normalizable solutions. But the retained terms in the solution Eq. (14) correspond to the solutions of the equations $\Box U\left(x\right)_\mu = 0$ and $\Box U\left(x\right)_{\mu_1 \mu_2} = 0$. Therefore, we may write the equations:

$$J = 1, \quad \Box\left(\Box + m_2^2\right)\left(\Box + m_3^2\right) \dots \left(\Box + m_N^2\right) U\left(x\right)_\mu = j\left(x\right)_\mu, \tag{45}$$

$$J = 2, \quad \Box\left(\Box + m_2^2\right)\left(\Box + m_3^2\right) \dots \left(\Box + m_N^2\right) U\left(x\right)_{\mu_1 \mu_2} = j\left(x\right)_{\mu_1 \mu_2} \tag{46}$$

where the physical currents $j\left(x\right)_\mu$ and $j\left(x\right)_{\mu_1 \mu_2}$ must obey the theorem on currents and fields Eqs. (42), (43) as well as the theorem on current asymptotics (Eq. (44)).

The Eq. (45) can be used for the particles of the photonic kind ($J^p = 1^-,\ C = -1$). Possibly the Eq. (46) can be used to take into account the quantum effects in the interactions of the gravitonic kind of the particles ($J^p = 2^+,\ C = +1$). The fields corresponding to last the 2 -spin particles may be regarded as addition to the fields considered in the general theory of the relativity at short distances. It is due to the fact that the general theory of the relativity does not include the quantum effects, which are just important at short distances. As it is known the quantum effects are important at the atomic distances already. Indeed, it is well known, that accordingly to the classical physics the atom cannot exist more than 10^{-10} c. Therefore, we may assume that the quantum effects are important at the atomic distances (which correspond to the density of the water) and at shorter distances.

The Eqs. (45), (46) are similar to Eq. (9). Therefore we may assume that the solutions of Eqs. (45), (46) are continuous in any point of the space-time and the formulae similar to Eqs. (24), (35) – (38) are valid. Therefore, we may expect that the electromagnetic and the gravitonic interactions, ought to have the relaxation of the interactions at short distances, which is similar to asymptotic freedom. This is consequence of the higher degree of the differential equations (9) as well as necessary existence of massive photons and gravitons.

We may expect that the use of the Eqs. (25) , (45), (46) allows to eliminate the quadratic divergences in the triangular anomaly for the graviton-photon-photon interaction [14].

If we assume that the gravitational interactions are described by Eq. (46) then at short distances the interactions are relaxed (in accordance with eq. (38)) and we may conclude, for example, that the existence of the black holes becomes problematic. But the complete solution of the question on the existence of black holes is possible in the quantum theory of the gravitation.

KINDS OF ELEMENTARY PARTICLES

Consider the distribution of the elementary particles in the kinds (or the dynasties). For the photon and gluon we have that m_1 is equal to zero. Since for the particle of integer spin $N \geq 3$, two (or greater) massive members of the photon kind must exist. They must have zero electric charge, $J^P = 1^-$, $C = -1$. These particles must contribute to the amplitudes of the reactions $e^+e^- \to e^+e^-$, $e^+e^- \to \mu^+\mu^-$, $e^+e^- \to$ hadrons at high energies and lead to the resonance behavior. We can expect that the coupling constants for the interactions of these members of the photonic kind with the leptons and the hadrons of the same electric charges must be equal. Therefore, the vector mesons ρ, ω, φ, J/ψ cannot be the members of the photonic kind. Similarly, in the gluonic kind two (or greater) massive colored particles must exist. Besides, three (or greater) massive members must exist in the Z^0 - and $W^{\pm}$ - kinds. In relations with the necessary existence of massive photons and gluons such questions arise: 1) is the gauge invariance for massive photons and gluons possible or not? 2) is the scaling in deep inelastic lepton – hadron scattering at higher energies exist or not?

It has been shown that for the $\frac{1}{2}$ - spin particles the number of the kind members (i.e. generations) must be equal to 5 (or greater). We assume that there exist electronic kind ($e_1 = e,\ e_2 = \mu,\ e_3 = \tau,\ e_4,\ e_5,\ \ldots$), the neutrinic kind $\left(\nu_1 = \nu_e, \nu_2 = \nu_\mu, \nu_3 = \nu_\tau, \nu_4, \nu_5, \ldots\right)$, three kinds of the coloured *up*-quarks $\left(u_1 = u,\ u_2 = c,\ u_3 = t,\ u_4,\ u_5, \ldots\right)$, and three kinds of the coloured *down*-quarks ($d_1 = d,\ d_2 = s,\quad d_3 = b,\ d_4,\ d_5, \ldots$). Note that in our approach only one neutrino may be massless. The higher members of the electronic and quark kinds can decay. For example, e_4 and e_5 can decay into $e\nu\bar{\nu}$, $\mu\nu\bar{\nu}$ (similarly to $\mu \to e\nu\bar{\nu}$), and $\nu +$ hadrons. We can assume the possibility of radiative decays $e_4, e_5 \to \mu\gamma$ or $e_4, e_5 \to \mu\gamma\gamma$. We can expect that such interactions of higher Z^0 and $W^{\pm}$ will be fairly weak in comparison with the interactions of $Z_1^0 = Z^0$ (92.4 GeV) and $W_1^{\pm} = W^{\pm}$ (81 GeV), as consequence of large masses of higher Z^0 and $W^{\pm}$. We may assume that: i) Z_2^0 or Z_3^0 or $W_2^{\pm}$ or $W_3^{\pm}$ can interact with right currents; i i) the interactions of Z_2^0 or Z_3^0 with fermion may be determined by the mixing matrix similar to the Kobayashi – Maskawa matrix and Z_2^0 or Z_3^0 can induce the transitions between the fermions of different generations (like to the $s \to W_1^- u$ - transition). Therefore in addition to the investigations of the decays $Z_{2,3}^0 \to \mu^+\mu^- \mathrm{X}$ [2] it is of interest the study of the decays $Z_{2,3}^0 \to \mu^{\pm}e^{\mp}\mathrm{X}$, which are forbidden in the Standard Model. Note that in our approach the massive photons (γ_2, γ_3,.....) as well as $Z_2^0, Z_3^0, \ldots$ can decay into $\mu^+\mu^- X$. But the γ_2 and γ_3 cannot decay into $e^{\pm}\mu^{\mp}X$.

If higher neutrino, are enough heavy then fairly exotic decays $\nu_{4,5} \to e\mu\nu_{1,2}$ become possible.

Since for fermions $N \geq 5$ the Kobayashi-Maskawa matrix must have the fifth (or greater) order. This can be important for the effects of CP - violation.

Possibly the leptons and the quarks from the fourth and fifth generations can be observed in Fermilab or LHC.

CONCLUSION

We have shown that the integrals for the Green functions of the Klein-Gordon and Dirac equations diverge as they have different values at different fashions of the calculations. The partial differential equations of the degree greater than four must be considered to derive the Green functions with convergent integrals. The generalizations of the Klein-Gordon and Dirac equations have been proposed.

It has been shown that the Green functions of the proposed equations of the higher degrees are continuous functions, i.e., they have no any singularities in all the time – space. In particular, the interaction potentials have no singularities. It has been shown that in our approach the interaction forces must be proportional to the distances between particles at the short distances. We have derived that the interaction potentials must have the oscillatory form at short distances. It may be assumed that all interactions must be relaxed at short distances.

The solutions of the generalized homogeneous Klein-Gordon and Dirac equations are the sums of the terms corresponding to the different particles of different masses which belong to different generations.

The minimal number of the generations for bosons $N_b \geq 3$ and fermions $N_f \geq 5$. We may assume that these numbers are for kinds of the elementary particles such as electronic, neutrinic, *up*-quarks, *down*-quarks, photonic, Z^0 -, $W^{\pm}$ -, gluonic, gravitonic kinds (and possibly the kind including the Higgs boson). For the composite particles the minimal degrees of Eqs. (9) (25) can be great than $2N_b$ and N_f, respectively. We consider the degrees of these equations for the $\pi^{\pm}$ -mesons and the nucleous. As in the quark model $\pi^{\pm}$ -meson consists of the u -quark and d -

antiquark, we may assume that the each particle from the $\pi^{\pm}$-kind consist of one *up*-quark (u-, or c-, or t-quark, or u_4 or u_5,…) and one *down*-antiquark ($\bar{d}$-, or $\bar{s}$-, $\bar{b}$-antiquark, or $\bar{d}_4$, or $\bar{d}_5$,…).Thus the minimal degree of Eq. (9) for the particles from $\pi^{\pm}$- kinds is equal $2N$ ($\pi^{\pm}$- kinds)=$2N_f^2$ (i.e., the number of the generations in the $\pi^{\pm}$-kinds equals $N_f^2 = 25$).The $\pi^{\pm}$-kind include $\pi^{\pm}$-, K^{+}-, $D^{\pm}(1869)$-mesons.

As it is known the proton consists of two u-quark and one d-quark. Therefore, we assume that particles from protonic kind consist of two *up*-quarks (electric charge $Q = \frac{2}{3}$, where e is the proton charge) and one *down*-quark $\left(Q = -\frac{1}{3}\right)$. We begin from the consideration of one pair of u- and d- quarks. Then we can use the representation of the $SU(4, FS)$-symmetry. From the Pauli principle for three such quarks it follows the representation of $SU(4, FS)$-group must be symmetric with the component number $4\cdot5\cdot6/1\cdot2\cdot3 = 20$. The expansion of this representation with respect to the representations of the $SU(2, F)\times SU(2, S)$-group is given by $20 = 4\times4 + 2\times2$, which correspond to the Δ-isobar (the $\frac{3}{2}$-spin) and the nucleon (the $\frac{1}{2}$-spin). To derive the number of the generations in the protonic kind we consider the product of N_f (for *down*-quarks) times $N_f\left(N_f+1\right)/2$ (the number of symmetric states for two quarks from - quark kind). As result we derive N (proton kind)= N (neutron kind) $\geq N_f^2\left(N_f+1\right)/2$. In particular, we have N (proton kind)= N (neutron kind) ≥ 75 for $N_f = 5$. It is the number of the generations for the protonic kind and simultaneously it is the degree of Eq. (25). The protonic kind includes such known particles as p, $\sum^{+}(1189)$, $\Lambda_c^{+}(2285)$.

From the consideration of the particle kinds we may conclude that the classification of the quarks and the hadrons must be changed.

In our approach the propagators of the bosons and the fermions decrease as $\left(q^2\right)^{-N_b}$ and $\left(q^2\right)^{-N_f/2}$ as $\left|q^2\right| \to \infty$, respectively. Therefore, we may expect that the renormalizations will be finite in our approach. In addition the number of the coupling constants reduces, e.g., the interactions of all the particles from the pionic kind (25 or greater particles) with the particles from the protonic and the neutronic kinds (75 or greater particles) is described by one coupling constant (at known the quark masses).

However, to show these results the number of tasks and problems must be solved. In particular, the Lagrangians which lead to Eqs. (9), (25), (41), (45), (46) must be derived.

These Lagrangians can be used to obtain the energy-momentum tensors and to study the invariance under the gauge transformations for the particles from the photonic and gluonic kinds, which include massless particles (photon and gluons) and the massive particles. The S-matrix must be derived to calculate the reaction amplitudes. Besides, the known results of the Standerd Model (as well as the results of $SU(3,F)$-symmetry) must be reproduced in our approach.

Now we present same consequences of Eqs. (9), (25):

1. The massive photons (γ_2 or γ_3) or Z_2^0 or Z_3^0 can decay into $\mu^{+}\mu^{-}X$ observed in Ref. [2]. The investigations of the decays into $\mu^{\pm}e^{\mp}X$ may be of importance to separate the possible manifestations of massive photons and heavy Z^0 (Z_2^0 or Z_3^0, or…).
2. One neutrino may be massless and four (or greater) neutrinos must be massive.
3. The Kobayashi-Maskawa mixing matrix must have fifth (or greater) order. It may be important for the effects of the CP – violation.
4. In relation with necessary existence of heavy gluons the question on the scaling at higher energies can arise.
5. If higher neutrinos are enough heavy then the fairly exotic decay $\nu_{4,5} \to e^{\pm}\mu^{\mp}\nu_{1,2}$ may be possible.
6. In spite of necessary existence of massive photons the results developed in large distance physics (in such as the solid state physics, the plasma physics, the statistical physics, the atomic physics, the low energy nuclear physics) practically do not change.

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