Keywords: rail module; bimodal transportation; queueing system; fractal arrivals

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INVESTIGATION INTO THE BIMODAL TRANSPORTATION PROCESS BY MODELLING RAIL MODULE STATES

Summary. The bimodal transportation process, which takes into account the modelling of rail module states, has been studied. The article demonstrates marked graphs of rail module states with and without running gear change in operation. It has been established which states have the greatest impact on the probability of a steady mode. The work has considered fractality of arrivals and its range in the queueing system with priorities.

1. INTRODUCTION

Countries of Western Europe, the USA and Canada have increasingly used combined freight transportation. It is one of the most promising types of transportation which combines the best qualities of rail and road transport. In Ukraine and Eastern Europe (Poland, Slovakia, Hungary, Lithuania, Latvia, Belarus and others) combined transportation does not have enough development; thus, there is a need to study the possibility of a more widespread introduction of such transportation.

2. LITERATURE REVIEW AND STATEMENT OF THE PROBLEM

A great number of scientists have been engaged in the issue of improving the mixed, combined and intermodal transportation by examining a wide range of problems.

Paper [1] emphasizes the fact that the European transport policy, within the framework of the flagship initiative “An efficient Europe in terms of resource use”, at present, is aimed at creating a system to support the European economic progress so as to improve the competitiveness and provide high-quality mobility, while ensuring more effective use of resources through intermodal transportation. The work considers the issue of pricing in close relation with freight transportation, starting from motor vehicles, switching to rail transport and ending with motor vehicles once again, without covering freight transshipment technology.

Papers [2] and [3] consider the technology of attaching semi-trailers to freight trains, performed by one person (the driver of the truck tractor) on a special platform without any additional handling devices. To perform these operations, only the pneumo-lifters that the trailers are equipped with are used, while the time required for a semi-trailer to be included in or excepted from freight trains does not exceed 6–10 minutes, but the question regarding mathematical substantiation of the bimodal freight transportation technology remains unresolved.

In developed countries logistics and intermodal transportation have become major factors for economic development, spatial communication and market integration. International traffic flows are practically not considered without intermodal technologies, namely, a network of logistics centers and intermodal transport terminals, as stated in studies [4] and [5]. However, to develop such a system, the technical equipment of these terminals and operational procedures should be considered more thoroughly.

Article [6] deals with the prospects for combined rail and road freight transportation as a single logistics transport chain. It also gives special attention to the current situation in Europe in terms of such transportation, considers technical limitations and the possibilities for transshipment, besides some solutions on how to improve service to consignors.

The authors of study [7] propose ways to improve the infrastructure of terminals intended for freight transshipment from rail to road or sea transport in order to maintain or even heighten their competitiveness in the freight transportation market and to increase effectiveness of the whole network.

Paper [8] is concerned with a new automated container terminal – the system for container transshipment from rail tracks to marine terminals or trucks by using ground trolleys, transfer platforms and frame trolleys. The research divides the system into several subsystems. By using a Markov chain the model for analyzing freight capacity of transfer platforms has been developed, and numerous tests have been conducted regarding the processing speed of ground trolleys.

The experience of some countries has proved the economic efficiency of intermodal freight transportation [9] by emphasizing its door-to-door freight delivery, though its main disadvantages are long periods of freight loading and shipment and the absence of mathematical substantiation for the transportation technology.

Paper [10] studies the rail transportation technology with subsequent transshipment to road transport in a bimodal transportation system, but the marked graphs of states do not consider indicators of transport means with respect to their waiting time or their yard time, though in practice it takes a lot of time and there is a need to consider these indicators.

Thus, an analysis of studies devoted to bimodal freight transportation has shown that the issue of its effective implementation in Ukraine and Eastern European countries as well as in the formation of complex implementation procedures has not been sufficiently studied yet.

3. OBJECTIVE AND RESEARCH TASKS

The objective of the work is to investigate combined freight transportation. The directions of efficiency of interaction between road and rail transport can be determined by modelling of rail modules state (with or without change of the running part in operation). Thus, there is a need to create a complex procedure for bimodal transportation.

To achieve the objective, the following tasks have been set:

- to analyze the current state of combined transportation in Ukraine and Eastern European countries;
- to build marked graphs of states and form a system of differential equations based on the graphs of rail module states according to different technologies; and
- to determine a usable range of the Poisson flow of arrivals.

4. MATHEMATICAL MODELLING IN RAIL MODULE OPERATION UNDER BIMODAL TRANSPORTATION

Bimodal transportation combines the advantages of road and rail transport. Due to a special design and the adaptation of semi-trailers they can be used for both road and rail (as an element of freight trains) transportation. Transshipment is carried out with (Fig. 1) or without change of the running gear, which is stipulated by the requirements for bimodal transportation [11]. This method of transport operation can shorten load/unload times, thus improving efficiency, providing cost-effective delivery by avoiding “dead weight” (car’s moving part that always goes with the wagon’s body) and facilitating door-to-door mobility at high efficiency of mobile units.

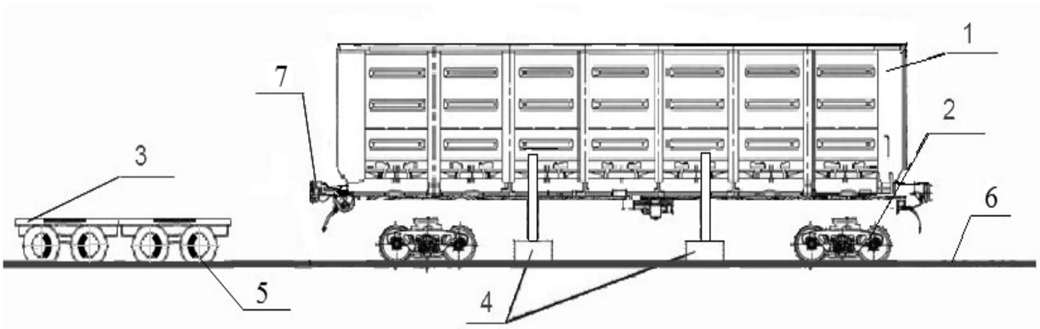

Fig. 1. Change from rail running gear to truck running gear on the extended jacks: 1 – open wagon; 2 – rail running gear; 3 – truck running gear; 4 – jacks; 5 – air brakes; 6 – railhead level; 7 – rotating automatic coupling device of the wagon

Bimodal transportation solves one of the most important engineering problems regarding fast transition from the 1520mm gauge to the 1435mm gauge on international routes. Non-transshipment transportation technology has become crucial for transportation of dangerous cargoes not subject to transshipment.

The widespread introduction of combined transportation may reduce competition in rail and road transport in the sector of wagon and group shipments. Improved transportation technology of only one type of transport, such as rail (increased flexibility of rail operations, which takes into account changes in wagon-flow formation [12]), will not considerably change the situation.

When using semi-trailers of the bimodal design, which meet the standards of Ukraine’s transport network formation, it is advisable to implement approaches to determine the commercial compatibility of the rolling stock as stated in [13].

According to Pearson’s chi-squared test the flow of arrivals is exponential: i.e., the Poisson flow (the simplest, with no after-effects, ordinary). This allows building graphs of railway module states and making up systems of the Kolmogorov differential equations. It is appropriate to consider the network of states in mathematical models as a queueing system.

4.1. Formation of the bimodal transportation procedure by using a rail module with running gear change in operation

A model of the bimodal freight transport technology with running gear change in operation can be presented as follows (Fig. 2).

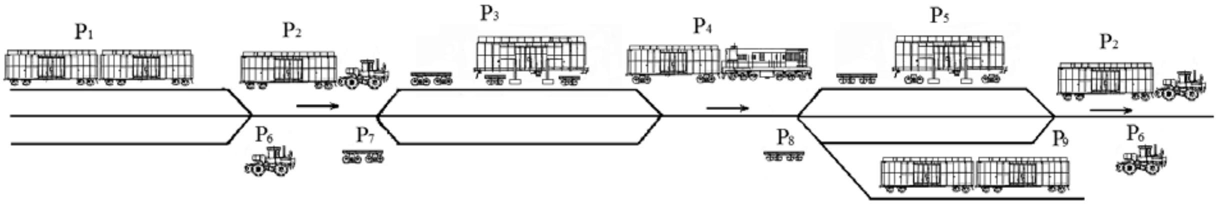

Fig. 2. Organization of the bimodal freight transportation technology with running gear change in operation

The marked graphs of states and the calculated probability of states of freight modules with running gear change in operation:

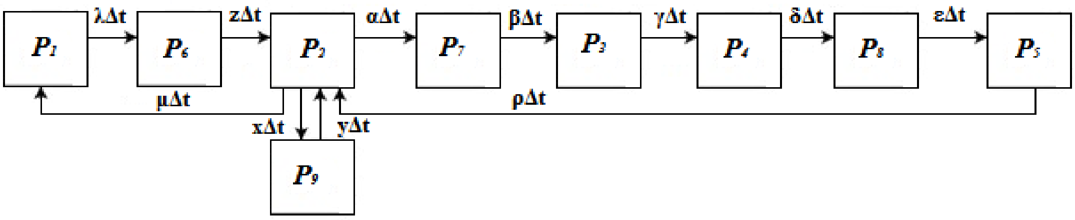

Fig. 3. The marked graph of the rail module states with running gear change in operation

According to the graph of the module states with running gear change in operation (Fig. 3) the following symbols have been introduced:

$P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9$ correspond to state probabilities of the rail module in traffic elements as shown in Table 1;

$\lambda, \mu, \alpha, x, \gamma, \delta, \rho, z, \beta, \varepsilon, y$ correspond to transition intensity from different states, respectively; the values are given in Table 2.

Table 1

Probabilities of rail module states in traffic elements

State of the transport vehicle	Probability of the state
In freight operations	P_1
In road freight transportation	P_2
Change from truck running gear to rail running gear at the departure station	P_3
Freight transportation by rail	P_4
Change from rail running gear to truck running gear at the destination station	P_5
Waiting for a truck tractor	P_6
Waiting for rail running gear	P_7
Waiting for truck running gear	P_8
Yard time of freights	P_9

Table 2

Intensity of functioning states for the rail module

Intensity of the state	Value of the state probability according to the technology		Probability of the state	
	With running gear change	Without running gear change	Functioning	Non-functioning
$\lambda = \frac{1}{T_o}$	0.29	0.29	$\lambda\Delta t$	$1-\lambda\Delta t$
$z = \frac{1}{T_{or}}$	0.28	0.27	$z\Delta t$	$1-z\Delta t$
$\mu = \frac{1}{T_{co}}$	0.15	0.15	$\mu\Delta t$	$1-\mu\Delta t$
$\alpha = \frac{1}{T_{wr}}$	0.23	0.23	$\alpha\Delta t$	$1-\alpha\Delta t$
$\beta = \frac{1}{T_{ca}}$	0.25	0.26	$\beta\Delta t$	$1-\beta\Delta t$
$\gamma = \frac{1}{T_r}$	0.15	0.16	$\gamma\Delta t$	$1-\gamma\Delta t$
$\delta = \frac{1}{T_{wa}}$	0.27	0.26	$\delta\Delta t$	$1-\delta\Delta t$
$\varepsilon = \frac{1}{T_c}$	0.18	0.21	$\varepsilon\Delta t$	$1-\varepsilon\Delta t$
$\rho = \frac{1}{T_a}$	0.13	0.15	$\rho\Delta t$	$1-\rho\Delta t$
$x = \frac{1}{T_d}$	0.10	0.10	$x\Delta t$	$1-x\Delta t$
$y = \frac{1}{T_z}$	0.10	0.09	$y\Delta t$	$1-y\Delta t$
$\nu = \frac{1}{T_{cr}}$	–	0.22	$\nu\Delta t$	$1-\nu\Delta t$
$\kappa = \frac{1}{T_{tr}}$	–	0.18	$\kappa\Delta t$	$1-\kappa\Delta t$

The intensities for the marked graph of the rail module states with the running gear change in operation have been determined and the system of differential equations has been formed as follows:

$$\begin{cases} \dfrac{dP_1}{dt} = -\lambda P_1 + \mu P_2; \\ \dfrac{dP_2}{dt} = -(\mu + \alpha + x)P_2 + \rho P_5 + zP_6 + yP_9; \\ \dfrac{dP_3}{dt} = -\gamma P_3 + \beta P_7; \\ \dfrac{dP_4}{dt} = \gamma P_3 - \delta P_4; \\ \dfrac{dP_5}{dt} = -\rho P_5 + \varepsilon P_8; \\ \dfrac{dP_6}{dt} = \lambda P_1 - zP_6; \\ \dfrac{dP_7}{dt} = \alpha P_2 - \beta P_7; \\ \dfrac{dP_8}{dt} = \delta P_4 - \varepsilon P_8; \\ \dfrac{dP_9}{dt} = xP_2 - yP_9. \end{cases} \quad (1)$$

It is advisable to present the normalization of the formed system of differential equations (1) as follows: $\sum P_i = 1$.

Thus, the steady state P_1 can be determined as

$$P_1 = \frac{1}{1 + \left[\frac{\lambda}{\mu}\left(1 + \frac{x}{y}\right) + \frac{\alpha\lambda}{\mu}\left(\frac{1}{\gamma} + \frac{1}{\delta} + \frac{1}{\rho} + \frac{1}{\beta} + \frac{1}{\varepsilon}\right) + \frac{\lambda}{z} \right]}. \quad (2)$$

The given freight transportation technology [11] provides for operations with a rail module with or without running gear change, the latter is used when the goods' owner can provide a spur track.

4.2. Formation of the bimodal transportation procedure by using a rail module without running gear change in operation

A model of the bimodal freight transportation technology without running gear change in operation can be presented as follows (Fig. 4).

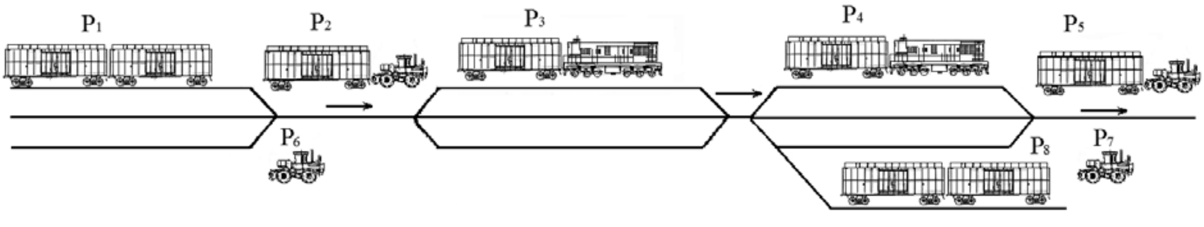

Fig. 4. Organization of the bimodal freight transportation technology without running gear change in operation

As for the above-mentioned variant, let us present probabilities of rail module states without running gear change in operation in the form of a marked graph (Fig. 5).

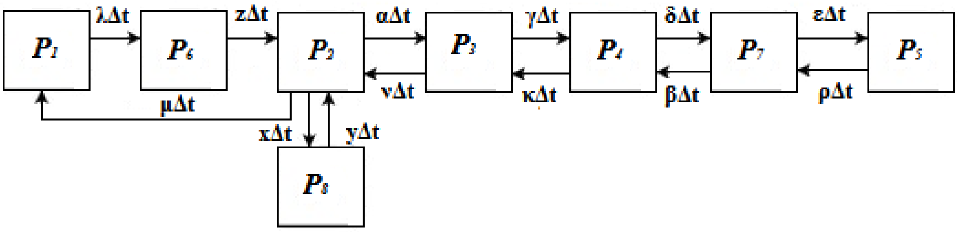

Fig. 5. The marked graph of rail module states without running gear change in operation

According to the graph of rail module states without running gear change in operation (Fig. 5) the following symbols have been introduced:

$P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8$ correspond to the probability of rail module states in traffic elements; the values are shown in Table 3;

$\lambda, \mu, \alpha, x, \gamma, \delta, \rho, z, \beta, \varepsilon, v, k, y$ correspond to the intensity of transition from different states; the values are given in Table 3.

Table 3

Probabilities of rail module states in traffic elements

State of the transport vehicle	Probability of the state
In freight operations	P_1
Freight delivery by truck tractor without running gear change by rail from the consignor	P_2
Freight transportation by rail to the departure station	P_3
Freight transportation by rail to the destination station	P_4
Freight delivery by truck tractor without running gear change by rail to the consignee	P_5
Waiting for the truck tractor at the consignor’s	P_6
Waiting for the truck tractor at the consignee’s	P_7
Yard time of freights	P_8

According to the graph of states the system of differential equations is as follows:

$$\begin{cases} \frac{dP_1}{dt} = -\lambda P_1 + \mu P_2; \\ \frac{dP_2}{dt} = -(\mu + \alpha + x)P_2 + \nu P_3 + zP_6 + yP_8; \\ \frac{dP_3}{dt} = (\mu + \alpha + x)P_2 - \gamma P_3 + \delta P_4; \\ \frac{dP_4}{dt} = \gamma P_3 - (\delta + \kappa)P_4 + \beta P_7; \\ \frac{dP_5}{dt} = -\rho P_5 + \varepsilon P_7; \\ \frac{dP_6}{dt} = \lambda P_1 - zP_6; \\ \frac{dP_7}{dt} = \delta P_4 + \rho P_5 - (\beta + \varepsilon)P_7; \\ \frac{dP_8}{dt} = xP_2 - yP_8. \end{cases} \tag{3}$$

It is advisable to present the normalization of the formed system of differential equations (3) as follows: $\sum P_i = 1$.

Thus, the steady state P_1 is

$$P_1 = 1 \Big/ \left(1 + \frac{\lambda}{\mu} + \frac{\lambda}{v}\left(\frac{\mu+\alpha+x}{\mu} - 1 - \frac{x}{\mu}\right) - \frac{\lambda}{\delta}\frac{\mu+\alpha+x}{\mu} + \frac{\lambda}{v}\frac{\gamma}{\delta}\left(\frac{\mu+\alpha+x}{\mu} - 1 - \frac{x}{\mu}\right) - \right.$$
$$\left. -\frac{\varepsilon\lambda}{\rho}\left(\frac{\mu+\alpha+x}{\mu\beta} + \frac{\gamma}{\delta\beta v}\left(\frac{\mu+\alpha+x}{\mu} - 1 - \frac{x}{\mu}\right)\right) + \frac{\lambda}{z} - \frac{\mu+\alpha+x}{\mu}\frac{\lambda}{\beta} + \frac{\gamma\lambda}{\delta\beta v}\left(\frac{\mu+\alpha+x}{\mu} - 1 - \frac{x}{\mu} + \frac{\lambda}{\mu}\frac{x}{y}\right)\right). \tag{4}$$

The solution to the system of differential equations for the input parameters ($\lambda, \mu, \alpha, x, \gamma, \delta, \rho, z, \beta, \varepsilon, v, k, y$) was computerized in the MathLAB program. The average periods for appropriate states have been chosen as the output data; they have been chosen according to the model, the marked graph of which is presented in Fig.3 and Fig. 5.

According to the computed results the curves $P = f(t)$ have been built, as given in Fig. 6 and Fig. 7.

The study into bimodal transportation by rail for systems (1) and (3) is demonstrated in Fig. 6 and Fig. 7, which testify that the following factors influence the probability of a steady state:

– according to Fig. 6: P_3 is the probability of change from truck running gear to rail running gear at the departure station, P_5 is the probability of change from rail running gear to truck running gear at the destination station, P_7 is the probability of waiting for rail running gear, P_8 is the probability of waiting for truck running gear;

– according to Fig. 7: P_3 is the probability of freight transportation by rail to the departure station, P_5 is the probability of freight delivery with a truck tractor without running gear change by rail to the consignee, P_7 is the probability of waiting for the truck tractor at the consignor's, P_8 is the probability of staying in the yard.

The considered flow of arrivals is the Poisson flow, but this assumption should have some limits in use. Thereafter, for this purpose, we consider fractal arrivals and find their usable range.

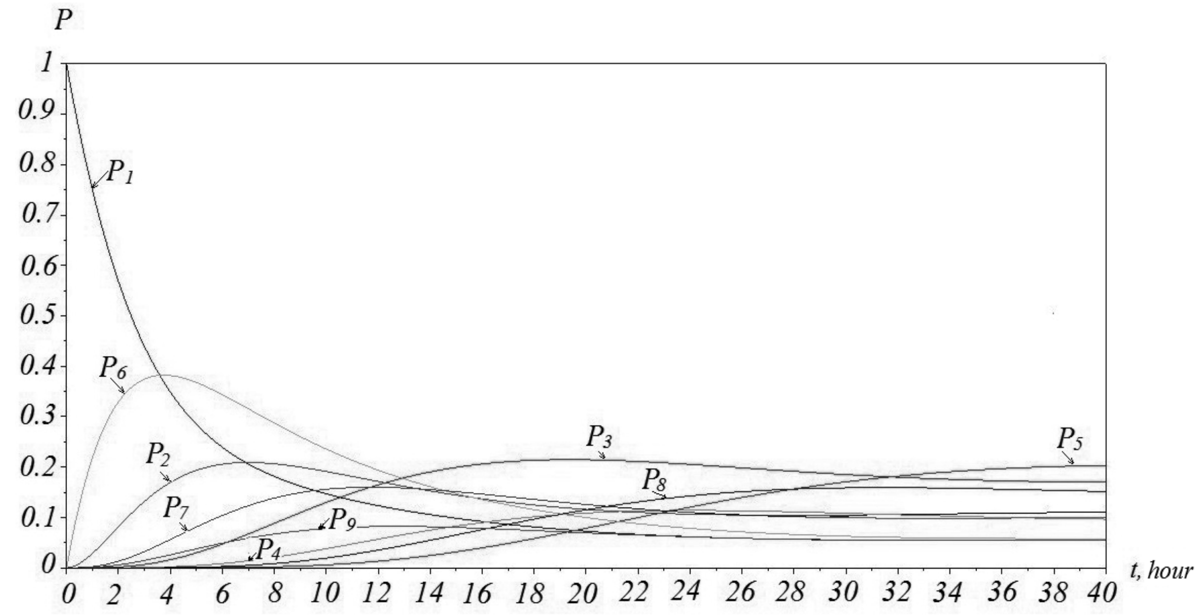

Fig. 6. The dependency of probabilities of the rail module states with running gear change in operation on service time

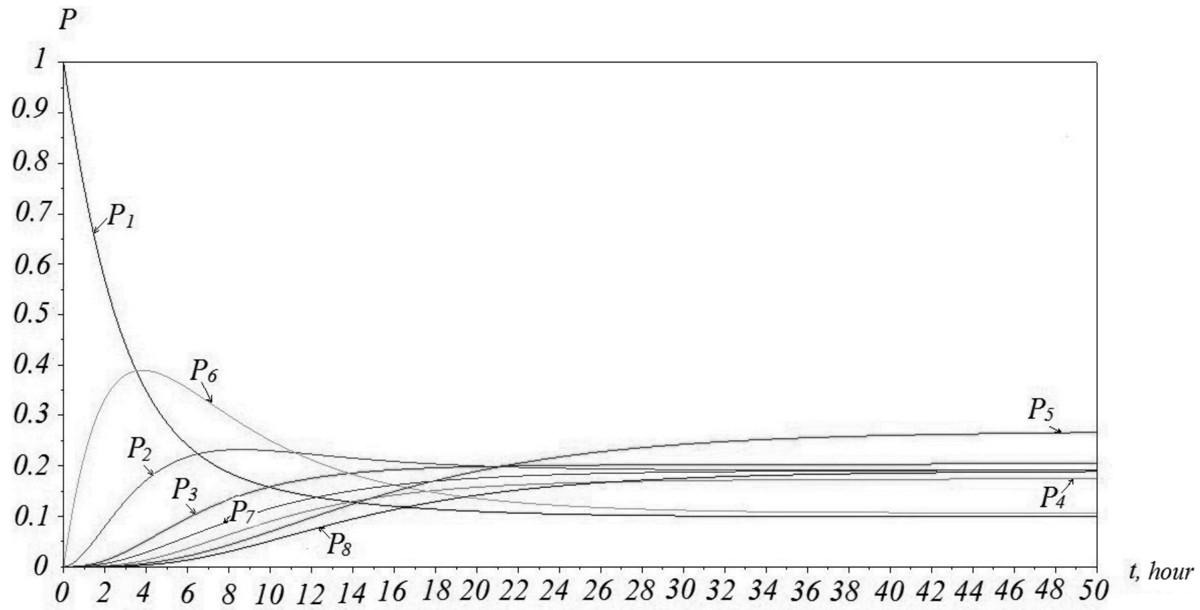

Fig. 7. The dependency of probabilities of the rail module states without running gear change in operation on service time

5. CONSIDERATION OF FRACTAL ARRIVALS IN ANALYZING THE QUEUEING SYSTEM WITH PRIORITIES

While considering the queueing system with priorities, it is a common practice to assume that the flow of arrivals is the Poisson flow, which does not agree with a fractal flow. Thus, while developing a new queueing model with priorities, it is necessary to consider the fractality of arrivals [14].

For the queueing system with the Poisson incoming flow requirements $(M / G / 1)$, the waiting time for the flow with the priority $p(T_{w_p})$ can be determined as:

$$T_{w_p} = \frac{\overline{T_{del}}}{(1-\sigma_{p-1})\cdot(1-\sigma_p)}, \tag{5}$$

where $\overline{T_{del}}$ is the average delay in requirements due to another service requirement;

σ_p is the variance of service time.

The value of this delay shall be determined by the formula

$$\overline{T_{del}} = \sum_{i=1}^{p}\overline{T_{del_i}} = \sum_{i=1}^{p} p_i \cdot \frac{\sigma^2_{\mathrm{b_i}}}{2\cdot\overline{T_{s_i}}} = \sum_{i=1}^{p}\frac{\lambda_i\sigma^2_{\mathrm{b_i}}}{2}, \tag{6}$$

where $\overline{T_{del_i}}$ is the average requirement delay due to the i[th] priority arrivals;

$\overline{T_{s_i}}$ is the average service time;

λ_i is the intensity of requirements of the i[th] priority arrivals, $\lambda_i = \dfrac{T_{s_i}}{P_i}$;

p_i is the loading coefficient of the i[th] priority requirements;

$\sigma^2_{\mathrm{b_i}}$ is the second moment in the service time (dispersion) of the i[th] priority requirements.

The Weibull distribution is most commonly used in modelling the fractal flow

$$F(x) = 1 - e^{-\left(\frac{x}{\beta}\right)\alpha}, \tag{7}$$

where β and α are the scale and form parameters, respectively.

Models of flow with long-term dependency lead to an asymptotic distribution of probabilities for the tail areas of the Weibull type, that is,

$$P(x > B) \sim e^{-(\gamma B)^{2-2H}} \text{ under } B \to \infty, \tag{8}$$

where γ is the constant;

$P(x > B)$ is the probability of the fact that the parameter x (e.g., Queue length) is more than the parameter B;

H is the Hurst exponent (self-similarity parameter).

The Hurst exponent H is the measure of stability of a statistical phenomenon or the measure of the length of a long-term dependency; the value $H = 0.5$ indicates the absence of long-term dependency. The closer the *H* value to 1, the higher the degree of stability of long-term dependency is. The Hurst exponent for most cases is in the range $0.5 < H < 1$ [15].

From expressions (7) and (8) it can be determined that the parameter α of the Weibull distribution can be expressed via the Hurst exponent as follows: $\alpha = 2 - 2H$.

Thus, while studying the queueing system $(G / G / 1)$ with priorities and fractal incoming traffic (flow), the parameter α of the Weibull distribution will be within the range $0 < \alpha < 1$.

By using the MathCAD program the dependencies $f(p_i) = \dfrac{T_w}{T_s}$ of the relative waiting time on the flow intensities for the systems $M / G / 1$ and $G / G / 1$ with priorities have been built (Fig. 8 – 11).

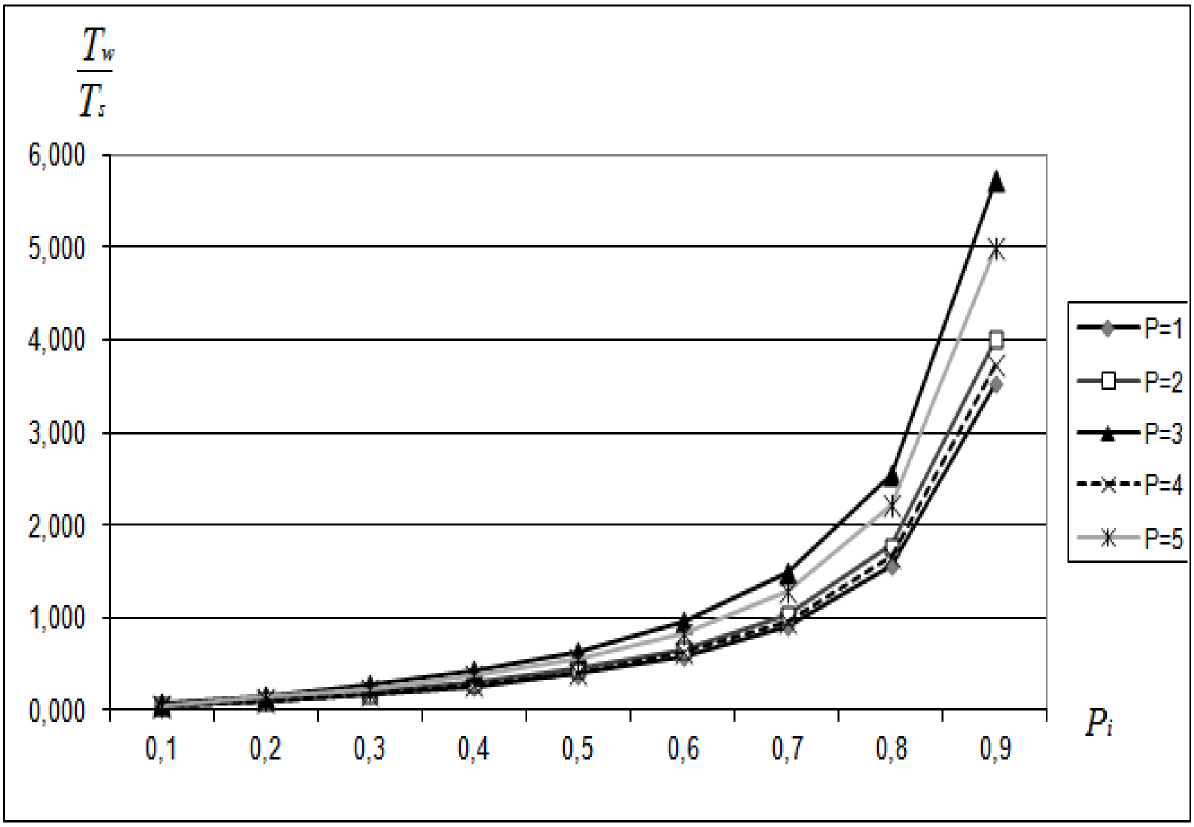

Fig. 8. The results of calculations for the queueing system $M / G / 1$ with priorities

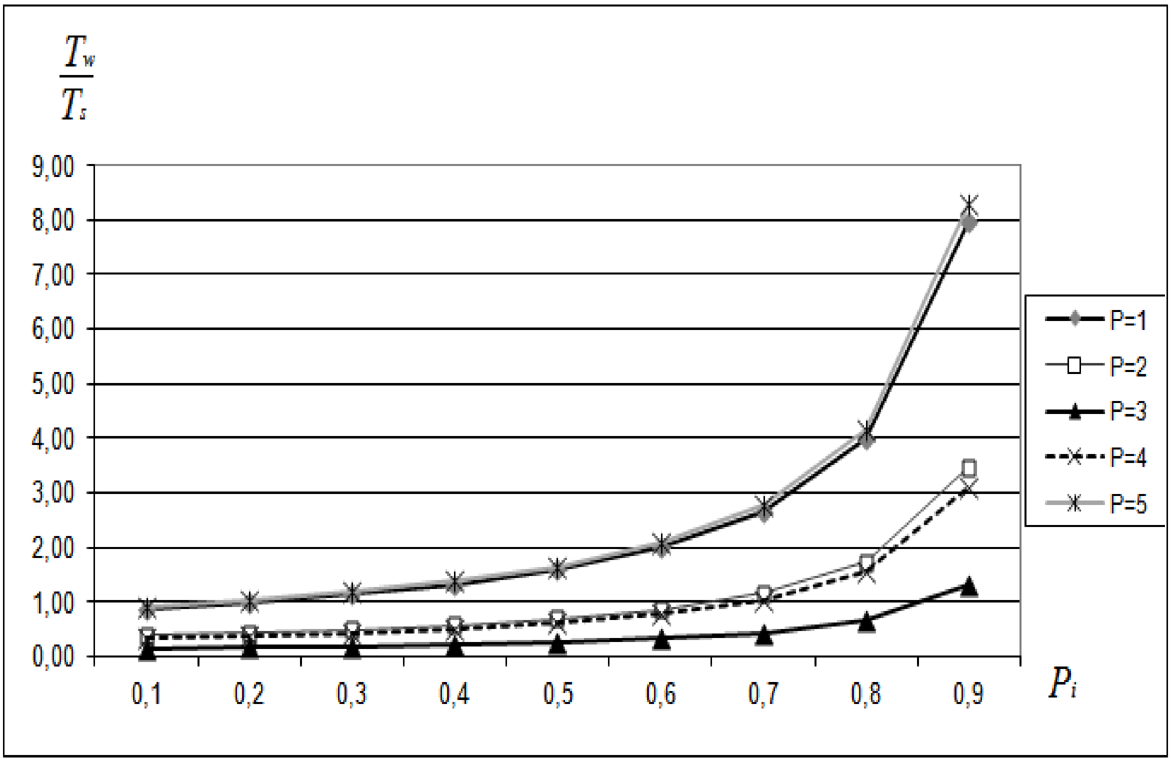

Fig. 9. The results of calculations for the queueing system $G / G / 1$ with priorities

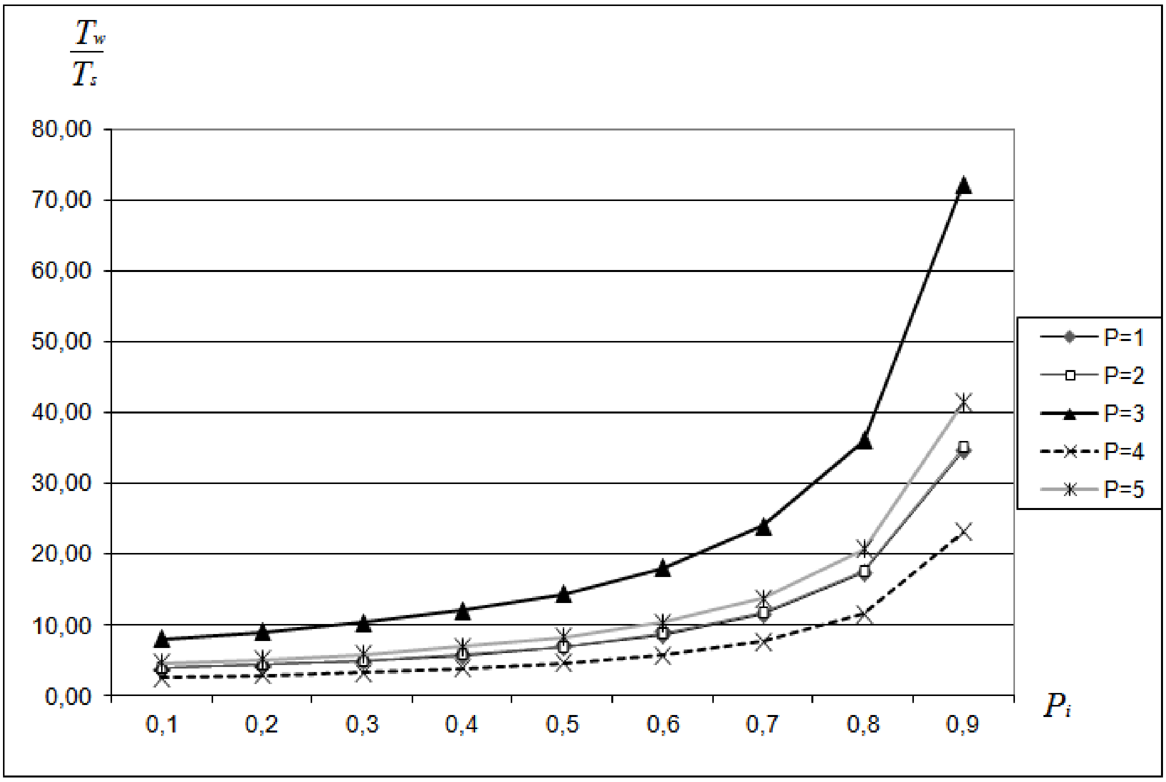

Fig. 10. The results of calculations for the queueing system $G/G/1$ with priorities at $H = 0.5$

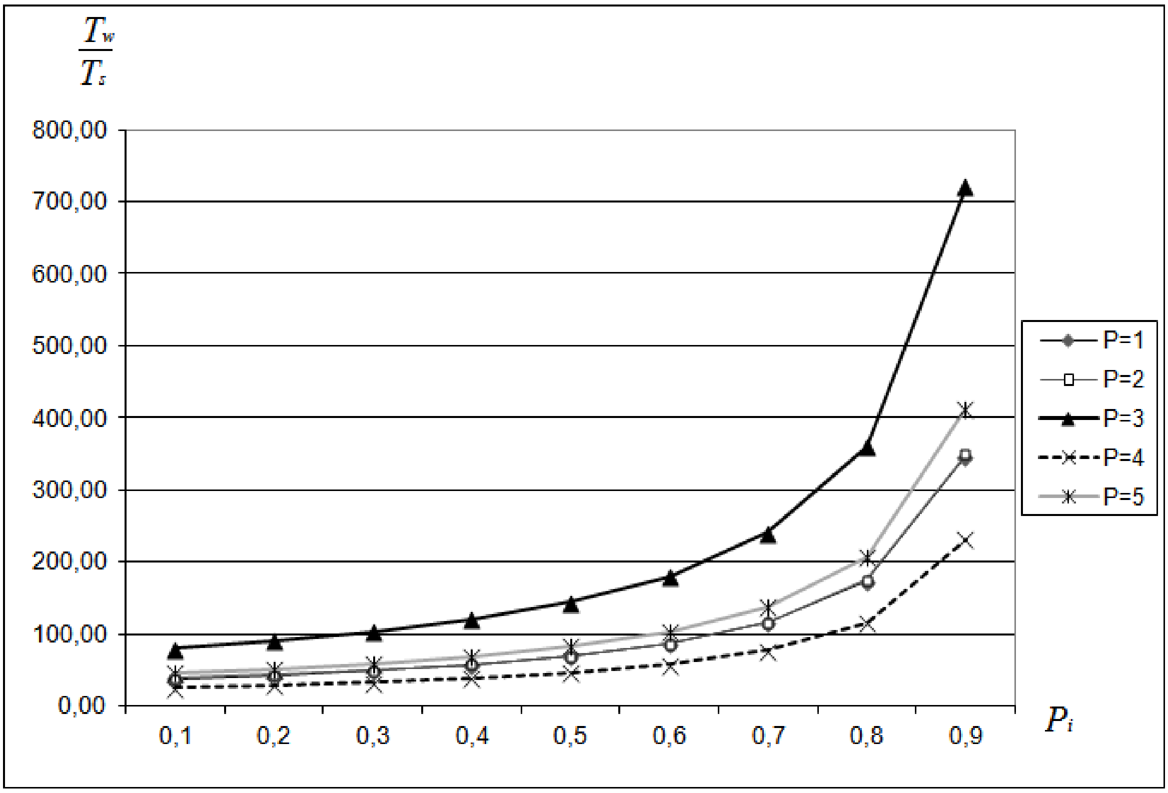

Fig. 11. The results of calculations for the queueing system $G/G/1$ with priorities at $H = 0.75$

According to the results of the built graphic dependencies ($f(p_i) = \frac{T_w}{T_s}$) it has been established that, in the range of the Hurst exponent $0.5 < H < 0.65$, one can calculate the appropriate parameters of the system as for the system $M/G/1$ with the Poisson flow of requirements; while the fractal input flow grows, there is a need to consider the service system with priorities in the form $G/G/1$.

The investigation presented in the article is a continuation of a series of research studies on this subject. As the issue of the efficiency of bimodal rail transportation has not been solved yet, further research on the subject is important and appropriate for the development of public transportation systems. The use of the findings obtained will enable transport companies (especially carriers) to use bimodal transportation more effectively and control primarily the rail module states of a higher influence on the stable regime probability, which is important for both the transportation process and for a higher service level to be provided to customers.

6. CONCLUSIONS

The investigation into the bimodal transportation process has demonstrated the following:

1. The analysis of the current state of combined transportation in Ukraine and Eastern European countries (Poland, Slovakia, Hungary, Lithuania, Latvia, Belarus and others) has proved that it is necessary to implement bimodal transportation on a wider scale, including those with advanced approaches, so as to determine the commercial effectiveness of the rolling stock for such transportation. These advanced approaches should include technologies for selecting freight wagons on a commercial basis during their distribution by the operating staff by applying a non-fixed decision-making system.
2. The marked graphs of states have been built, and corresponding systems of differential equations based on the graphs of rail module states by different technologies have been formed. On this basis it has been established that the steady mode probability is highly influenced by the change from rail running gear to truck running gear at the destination station, waiting for rail running gear, waiting for truck running gear, transportation of freight by rail to the departure station, delivery of freight by the truck tractor without running gear change by rail to the consignee’s, waiting for the truck at the consignee’s and the yard time.
3. By studying the provisions of fractal analysis, and the graphic dependencies obtained, it has been determined that under unstable characteristics of arrivals there is a need to consider the service system with priorities in the $G/G/1$ form; within the range of the Hurst exponent $0.5 < H < 0.65$, approximate parameters can be calculated as for the $M/G/1$ system; further, if the fractal traffic grows due to a significant difference in service system characteristics it is advisable to avoid traditional approaches.

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