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Study of influence of imprecision of primary information on energy consumption of rolling stock

The energy efficiency of urban rail transportation systems is a crucial indicator, as traction energy consumption typically accounts for 40-60% of the total energy consumption of the transportation system. This study examines the sensitivity of energy consumption to deviations from nominal conditions under the implementation of pre-calculated optimized trajectories for electric rolling stock, considering rolling stock with operation modes typical for suburban and urban transport. To determine globally optimal control strategies that minimize energy consumption while complying with operational constraints, the study uses dynamic programming based on Bellman's optimality principle. The optimization model divides the track section into discrete segments and uses the backward induction method to establish optimal control laws, producing speed trajectories as functions of the train's current coordinates on a given gradient profile. The trade-off between energy and time is represented by an indefinite Lagrange multiplier to ensure adherence to the timetable. Sensitivity analysis is performed by simulating inaccuracies in the estimates of the train's current coordinates and variations in its passenger load. Modelling of a targeted braking system has been implemented so as to ensure stopping accuracy in the event of measurement inaccuracies. Modelling was performed using three typical gradient profiles, characteristic primarily of underground railways; for comparison, modelling was also performed on a conditional section with a negligible gradient. The research methodology allows for a quantitative assessment of the degree of energy overconsumption that may be caused by deviations in train passenger load factors and errors in the estimation of the position of rolling stock (±25 meters), which provides information for assessing the effectiveness of pre-calculated optimized trajectories in real operating conditions.

***Keywords**: speed trajectory optimization, urban rail transport, energy efficiency, dynamic programming.*

Introduction. The reduction of energy consumption for a given traffic schedule is one of the most important priorities for urban rail transport systems. Electricity costs typically account for a significant portion of transport companies' operating costs, with traction energy accounting for approximately 40-60% of total energy consumption in such systems.

Speed trajectory optimization is one of the viable approaches for minimizing energy consumption through determining the optimal control strategy that minimizes power requirements. It relies on a systematic search for the most energy-efficient combinations of traction, coasting and braking phases, while satisfying schedule and safety constraints [1].

Dynamic programming, based on Bolman's optimality principle [2], is one of the most effective methods for optimizing speed trajectories due to its ability to guarantee convergence to a global minimum (within the margin of error of discretization). This method evaluates all possible control sequences by breaking down the optimizations problem into a series of sequential recursive decision-making steps, ensuring that the solution represents a truly optimal policy rather than a local minimum [3]. This mathematical basis provides a theoretical guarantee of optimality.

However, the dynamic programming approach requires a backward calculation of trajectories, starting from the final conditions and moving towards the initial state, which complicates real-time updates and requires preliminary calculations for each section of track under consideration [4]. In general, optimal trajectories must be calculated in advance for specific operating scenarios and saved for subsequent implementation by automated train control systems. Dependence on pre-calculated solutions becomes particularly problematic when considering that optimal trajectories are significantly affected by changes in train mass, since in modern metro rolling stock, passenger load can account for up to 40% of the total mass of the consist. These changes in weight directly affect the dynamic characteristics of the train and energy consumption.

The purpose and tasks of research. The main goal of this study is to quantitatively assess the sensitivity of energy consumption when operating on optimized trajectories, but with deviations from the calculated conditions. Although existing optimizations methods can generate globally optimal control strategies for specific scenarios, real-world driving conditions always involve a certain degree of uncertainty. The study aims to analyze the gap between theoretical optimizations and the practical application of calculated trajectories.

The study covers several interrelated tasks. First, to develop an algorithm for optimizing the train's trajectory under given route conditions, rolling stock characteristics and operational constraints. Second, to develop a model of rolling stock dynamics that allows for deviations of train characteristics from the calculated ones, while maintaining the optimized motion trajectory. Third, to conduct a systematic sensitivity analysis on different track profiles representing different operational complexities and to quantitatively assess energy overconsumption caused by load variations and spatial positioning errors within ±25 meters. Through this comprehensive approach, the study aims to provide a practical understanding of the reliability of pre-calculated optimal trajectories and to provide insights for decision makers on acceptable limits for operational tolerances that maintain energy efficiency benefits under real-world operating conditions in urban rail transport.

Analysis of recent research and problem statement. Numerous publications are devoted to the problem of electricity consumption in rail transport, particularly in the metro, and its optimization. General systematic approaches and reviews of energy-efficient control methods are presented in [5-7]. The main ways to reduce electricity consumption are to optimize the movement trajectory and/or optimize schedules. Research in this area covers the application of various mathematical methods, such as: Pontryagin's maximum principle [8-12], dynamic programming [1, 3-4, 13-16], as well as evolutionary, heuristic, intelligent and other optimization algorithms [17-22].

A separate group of works focuses on increasing the stability (robustness) of optimal trajectories to the influence of unpredictable factors, such as uncertainty of passenger load or changes in external conditions [26-27].

At the same time, despite significant achievements, the number of works devoted to the quantitative study of the impact of individual factors on the electricity consumption of rolling stock, in particular in the context of implementing pre-calculated optimal trajectories in conditions that deviate from the calculated ones, has been limited.

Research materials and methods. Trajectory optimisation for any vehicle is the process of finding the relationship between the vehicle's speed v and its coordinates s or time t that minimises (or maximises) a specific target value. As a rule, the task of speed trajectory optimisation is to find a trajectory $v(s)$ or $v(t)$ that would minimise the total energy consumption A for the movement of vehicle. Formally, in general, the task of optimization of the speed trajectory can be represented as:

$$\min_{V(\cdot)}\int_{0}^{t} A\big(v(t)\big)dt: \quad t \le T_3, \quad v(t) \le v_{max}, \quad ..., \tag{1}$$

where $A(v(t))$ is the energy consumption depending on speed trajectory $v(t)$, kWh;
$v(t)$ – speed trajectory as function of time, m/s;
t – time, s;
T_t – target time on route (scheduled), s;
v_{max} – maximum speed limited by traffic requirements or design speed of rolling stock, m/s.

To solve the optimization problem using dynamic programming, the continuous state x is divided into a discrete sequence. Let $J_i(x)$ be the minimum 'cost' from step i to the end of the grid, for state x, then the Bellman equation

$$J_i(x) = \min_{u}\left\{l_i(x,u) + J_{i+1}\left(f_i(x,u)\right)\right\}, \tag{2}$$

where l_i is a cost of singular step and f_i is a function of change of state given control sequence u.

In dynamic programming optimization, a transition table $J(x)$ and optimal control sequence $u(i)$ is calculated sequentially, moving backwards from the grid end. Resulting control sequence corresponds to a specific speed trajectory $x(i)$.

In this study, to optimize train speed trajectory, a given section of track with a length of S m is split into n separate independent segments, with train movement considered as a function of distance. The segments act as optimization steps. Each segment has a corresponding value of the gradient i, ‰. The segment length s_n, m, (discreteness of the division) is specified in advance before modelling and is selected to be sufficiently small (5-10 m) to reduce its impact on the accuracy of calculations. If a segment contains a gradient break, it is further divided into two smaller segments, thus fully preserving the longitudinal profile of the section.

The model used to optimize the train's trajectory (Figure 1) can be is of a network type [28] with feedback loops and consisting of two main structural components:

- the model of rolling stock dynamics (train movement model) that reflects the impact of control signal $u(n)$ on train movement, taking into account its load, traction characteristics and gradient conditions;
- the actual control and optimization model responsible for selection of the control signal $u(n)$ given the outputs of train movement model.

The criterion of optimality is reaching the final destination without exceeding the scheduled travel time for the route T_t, s, and with minimal total electricity consumption A, kWh.

The motion of a train is described by the basic equation of train motion

$$\frac{dv}{dt} = \frac{F - B - T}{P}\cdot\frac{g}{(1+\gamma)}, \tag{3}$$

where F is tractive effort, kN;
B – brake force, kN;
W – total resistance, kN;
P – train weight, kN;
g – gravitational constant (9,81 m/s^2),
γ – rotating mass inertial coefficient (≈0,12).

The weight of a train consists of the weight of the rolling stock itself and the weight of passengers

$$P = P_T + 0.7355 \cdot k_f \cdot C, \tag{4}$$

where P_T is the weight of the rolling stock (tare), kN;
0.7355 is average weight of a single passenger, kN (75 kg);
k_f is a coefficient of passenger load, 0…1;
C is maximum passenger capacity of a given train.

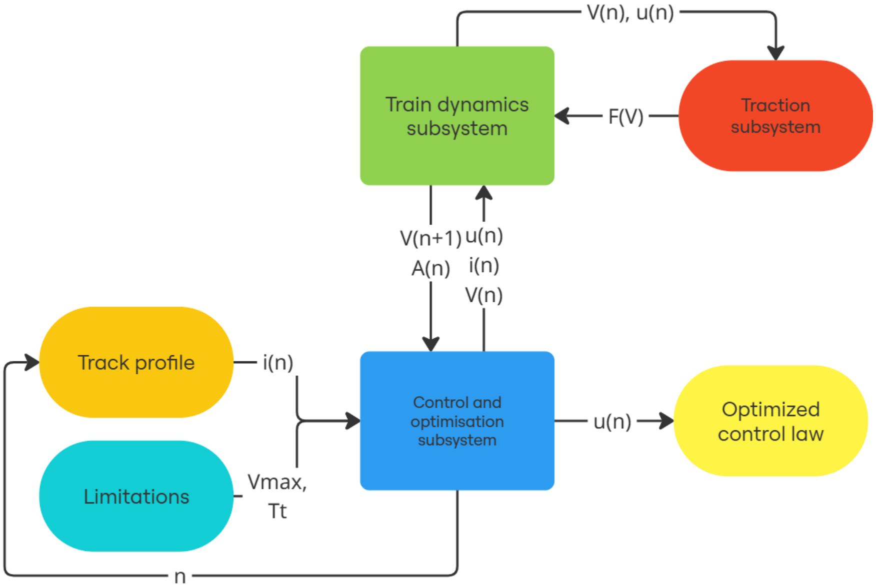

Fig. 1. **Structural diagram of the train trajectory optimization model**

The system of equations describing the operation of the train traction drive on the *n*-th track section is as follows:

$$F_n = \begin{cases} min\left[F_{max}, F\left(V\right)\right], & 0 < u \le 1; \\ 0, & u = 0; \\ B_{max} \cdot u_n, & -1 \le u < 0, \end{cases} \tag{5}$$

where F_n is tractive effort at *n*-th route section, kN;
F_{max} is maximum tractive effort of a motor car subject to current limitation or wheel-rail adhesion, kN;
B_{max} is maximum braking force of a motor car subject to current limitation or wheel-rail adhesion, kN (it was conditionally assumed that $B_{max} = -F_{max}$);
$F(V)$ is traction characteristics of a motor car depending on speed; V is speed, km/h;
u_n is a control signal (we hereinafter also refer to it as *traction application coefficient* for clarity) at *n*-th section.

For the purposes of this study, the traction characteristics and rolling stock parameters of the 81-7036/7037 model metro consist manufactured by PJSC 'Kryukiv Railway Car Building Works' were used for modelling. The 5-car consist is equipped with an asynchronous traction drive with a traction motor power of 4×180 kW per car.

The electricity consumption for passing any section of the route A_n, kWh, can be determined as follows during modelling

$$A_n = \begin{cases} \dfrac{\sum N_n \cdot \Delta t}{3600 \cdot \eta}, & F_n > 0; \\ 0, & F_n \le 0, \end{cases} \tag{6}$$

where N_n is total developed power of the train traction drive on the n-th section ($N_n = F_n \cdot V_n$);
η is total efficiency of the traction drive (for asynchronous drives η = 0,8…0,9; 0,85 was assumed for modelling), Δt_n is time to pass the n-th section;
3600 is conversion factor from seconds to hours.

With dynamic programming for each section of the route n all possible transitions between discrete states of the system are calculated. This means that for every possible section entry speed V_n calculations are performed sequentially for different values of the control signal u_n, whereby $u_n \in [-1, 1]$. For each combination (n, V_n, u_n) using equations 5 and 3 the resulting speed at the end of the section V'_n and electricity consumption A_n are determined. This also allows to determine the time it would take for a train to pass a single section t_n, s:

$$t_n = \frac{2 \cdot s_n}{\frac{V_n}{3.6} + \frac{V'_n}{3.6}}, \tag{7}$$

where s_n is a length of n-th route section, m;
3.6 – conversion factor from kilometres per hour to metres per second. Given the small size of the sections, the acceleration value on them is assumed to be constant.

Complete calculation of all valid combinations (n, V_n, u_n) allows the use of the recursive Bellman equation, which will include a similar equation for the next section $n+1$, i.e. the conditional cost value of passing through all subsequent sections

$$J_n(V_n, u_n) = \min_{-1 \le u_n \le 1} \left\{ A_n + \lambda \cdot t_n + J_{n+1}(V'_n) \right\}, \tag{8}$$

whereby the cost function of a single section n for control signal u_n is

$$l_n(V_n, u_n) = A_n + \lambda \cdot t_n, \tag{9}$$

where λ is indeterminate Lagrange multiplier and is determined iteratively.

After passing through all n sections in reverse order, a table of costs for all possible combinations is generated $J_n(V_n, u_n)$ by selecting for each section n such a u_n which corresponds to lowest cost function value J, moving backwards from the end state. Thus, an optimal control law for given conditions $u(s)$ is established.

The target trajectory is calculated for an average case with a passenger load coefficient of $k_f = 50\%$. In real conditions, the load coefficient is a volatile value that cannot be measured with high accuracy, so it is important to analyze how its deviation from the calculated value affects energy consumption, provided that the automated train operation system follows the calculated trajectory. Similarly, the current position of the train cannot be measured with perfect accuracy, so there are always certain deviations, which will also affect energy consumption on the route. The sources of error in determining the train's coordinates require further study, and the law of its change as a function of the distance travelled is unknown; therefore, for this study, it was assumed that the coordinate error is a constant value throughout the entire route.

Deviations in train weight and its current coordinates are entered into the motion simulation as follows. When performing traction calculations (acceleration and deceleration values), the ‘actual’ train weight P' calculated using equation 4 with arbitrary passenger load coefficient k_f is used. The train’s position s is substituted with fictitious position coordinate s'

$$s' = s + \Delta s, \tag{10}$$

where Δs is an error of determination of current train position, in meters.

To ensure accurate train stopping at stations with a small enough deviation of the stopping point from the target point, but with a high (and therefore energy-efficient) average deceleration value, a precision braking system has been introduced into the train movement simulation. A schematic diagram of this system is shown in Figure 2.

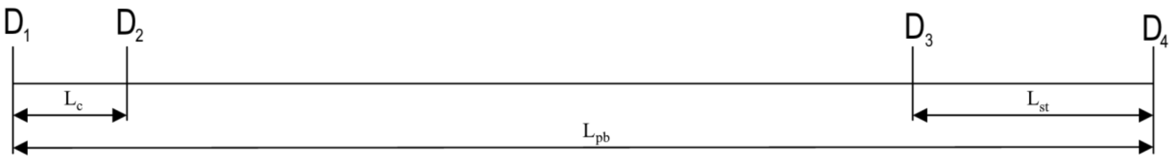

Fig. 2. **Layout of the train precision braking system**

The system consists of three sensors D_1, D_2 and D_3. D_4 indicates the designated stopping point of the train. The distance from the stopping point at which precision braking is applied is marked as L_{pb} and was established as 300 meters. When passing the first sensor D_1 the current train coordinate calculated by the onboard systems is reset and set equal to $S - L_{pb}$ m, i.e. the full length of the route except for the distance of targeted braking. On the segment between the first D_1 and second sensor D_2 a calibration of train coordinate measurements by onboard systems is performed by comparing them with a known distance L_c. This reduces the dependence of the braking trajectory on measurement errors. At a short distance from the stopping point L_{st} sensor D_3 again resets the current train coordinate, accepting it as equal to $S - L_{st}$. The target speed of the train in the precision braking zone is determined as

$$v(s) = 3.6 \cdot \sqrt{2 \cdot d \cdot (S - s)}, \tag{11}$$

where $v(s)$ is target train speed at coordinate s, km/h;
3,6 is conversion factor from m/s to km/h;
d is a target deceleration value during braking, м/с^2 (was established as 0,6 m/s^2);
S is the length of the segment, m; s is current train coordinate (distance from the starting point), m.

Modelling of such a system has shown that it is resistant to various forms of measurement errors (errors in speed measurement by an axial sensor, deviations in values L_{pb}, L_c, L_{st} from calculated) and ensures that the train stops with acceptable deviations from the stopping point [29].

To model train movement, three conditional types of gradient profiles were used: I (‘light’), II (‘medium’) and III (‘heavy’) developed on the basis of previous research findings [30]. They are presented in Tables 1-3. Additionally, for comparison purposes, calculations were also performed for a gradient profile with a constant gradient of 3‰ and a length of 1000 m (‘type 0’ gradient profile).

Table 1. **Type I conditional gradient profile (‘light’)**

Segment length, m	150	200	200	200	150	100
Gradient, ‰	3	10	-3	3	10	0

Table 2. **Type II conditional gradient profile (‘medium’)**

Segment length, m	150	50	150	50	100	250	400	50	100
Gradient, ‰	-5	-30	-30	-3	-3	-3	11	17	0

Table 3. **Type III conditional gradient profile ('heavy')**

Segment length, m	150	50	200	200	50	100	50	300	400	50	50	100
Gradient, ‰	5	-5	3	5	3	35	11	40	30	17	5	0

Examples of optimised speed trajectories and speed trajectories with measurement errors are shown in Figures 3-6. Solid lines correspond to calculated trajectories; dashed lines correspond to trajectories with errors.

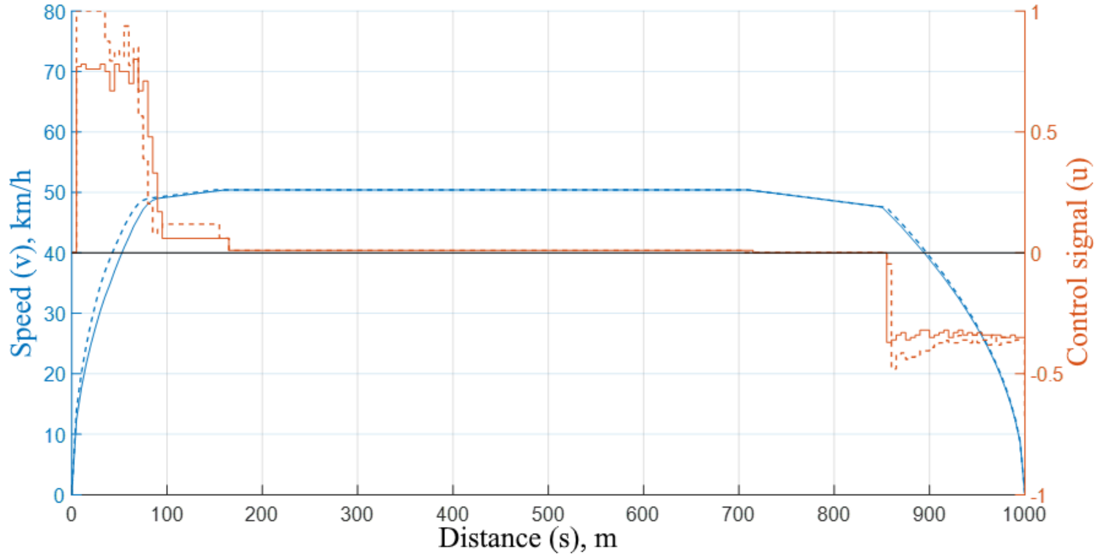

Fig. 3. **Example of train speed trajectories on a conditional gradient profile of type 0 [average speed – 40 km/h; actual passenger load coefficient – 1.0; coordinate estimation error – +10 m]**

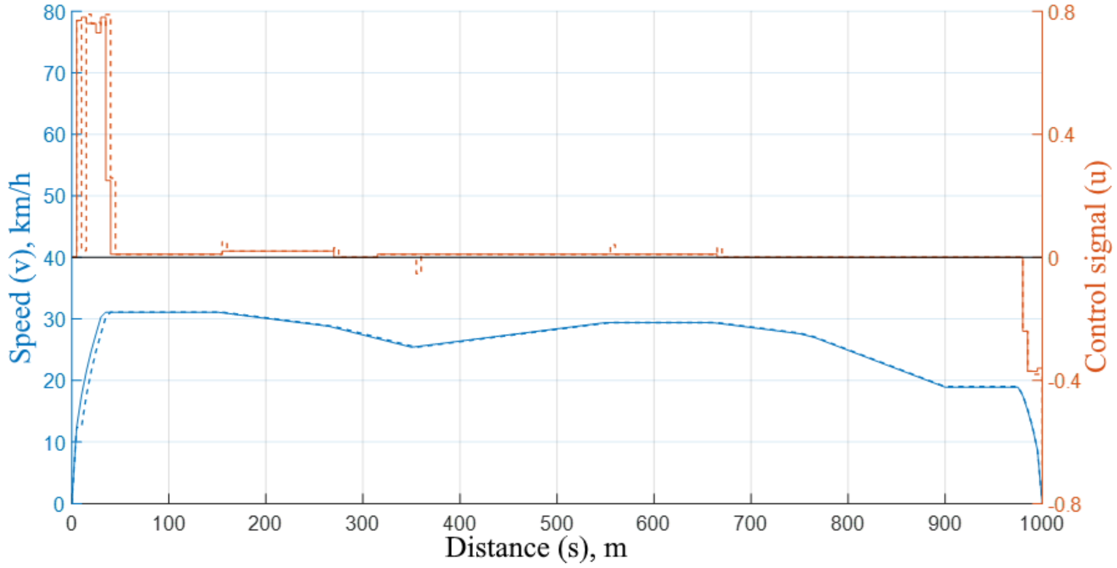

Fig. 4. **Example of train speed trajectories on a conditional gradient profile of type I ['light' profile; average speed – 25 km/h; actual passenger load coefficient – 0.2; coordinate estimation error – -5 m]**

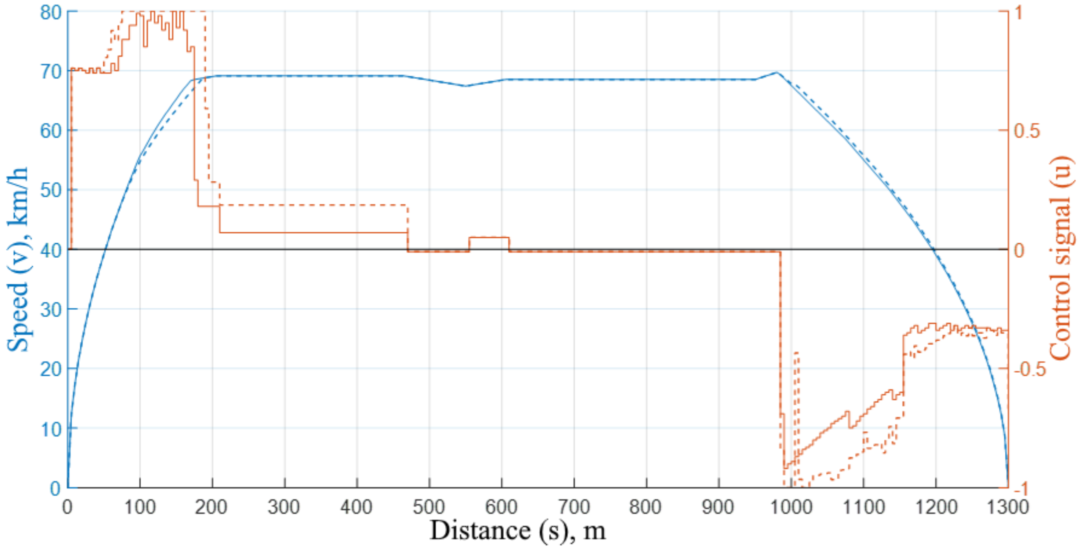

Fig. 5. **Example of train speed trajectories on a conditional gradient profile of type II ['medium' profile; reverse direction; average speed – 50 km/h; actual passenger load coefficient – 1.0; no coordinate estimation error]**

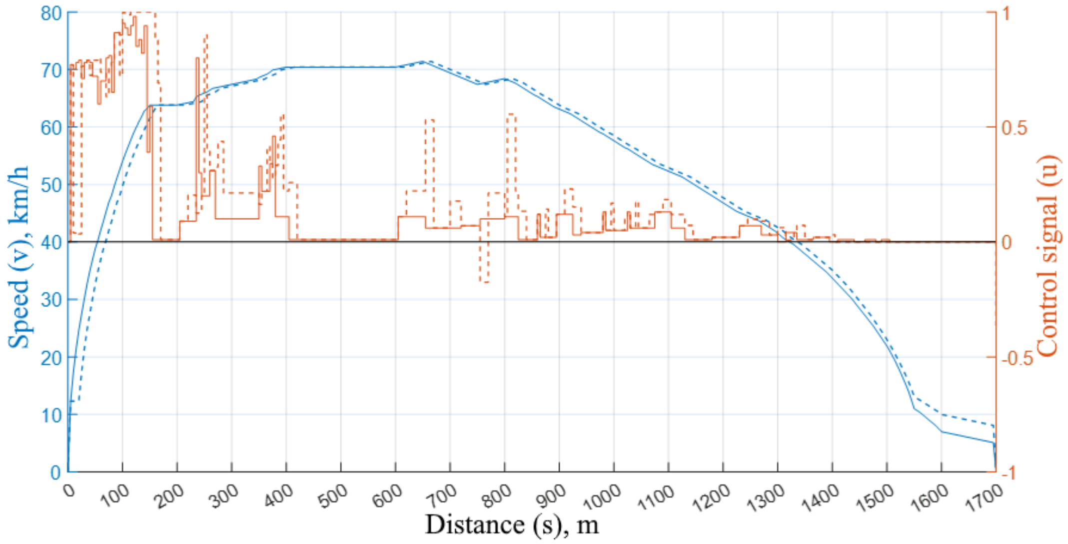

Fig. 6. **Example of train speed trajectories on a conditional gradient profile of type III ['heavy profile; average speed – 30 km/h; actual passenger load coefficient – 0.8; coordinate estimation error – -15 m]**

Any of the introduced errors in modelling the train's movement along a pre-optimized trajectory leads to an increase in the amount of electricity consumed to cover the route. Sensitivity analysis, in which different error values are entered into the system, allows to determine the impact of each of them on energy overconsumption. To perform the sensitivity analysis, a speed of 35 km/h was taken as the base average speed for the route (typical for metro operation); energy overconsumption ε_A is calculated relative to it as

$$\varepsilon_A = \frac{\left(A_f' + A_r'\right) - \left(A_f + A_r\right)}{A_f + A_r}, \quad (12)$$

where A_f, A_r are energy expenditures to cover the route along an optimised trajectory, in the absence of errors, in the forward and reverse directions, respectively;
A'_f, A'_r are energy expenditures to cover the route in the presence of train weight deviations and errors of measured coordinates, likewise in the forward and reverse directions.

Margins for calculation of energy overconsumption ε_A were established as follows: passenger load coefficient k_f from 0 to 1, train coordinate measurement error s' – from -25 to +25 м. The simulation summaries are presented in Tables 4-7 and Figures 7-10.

Table 4. **Energy overconsumption ε_A for 'type 0' gradient profile**

Passenger load coeff. / Coordinate error, m	0	0,25	**0,5**	0,75	1
25	2,2%	2%	1,9%	2,4%	3,5%
20	1,9%	1,8%	1,6%	2,2%	3,5%
10	2,0%	1,8%	1,7%	2,2%	3,2%
0	0,5%	0,1%	**0,0%**	0,5%	2,0%
-10	2,2%	2,1%	2,1%	2,3%	3,4%
-20	2,3%	2,2%	2,2%	2,3%	3,7%
-25	2,8%	2,8%	2,8%	2,9%	4,3%

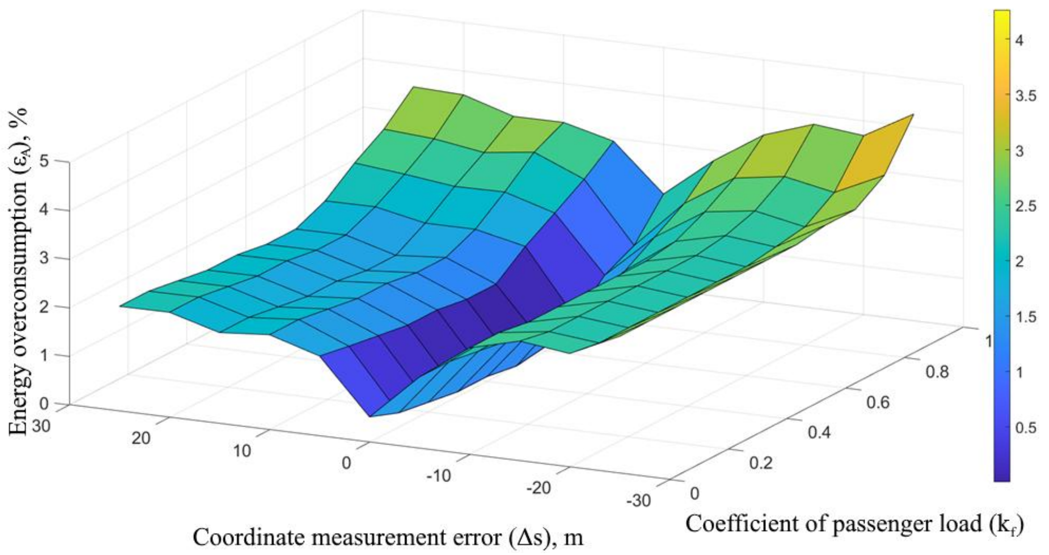

Fig. 7. **Dependency of energy overconsumption ε_A on error of measurement of train coordinate and train weight deviations for 'type 0' gradient profile**

Table 5. **Energy overconsumption ε_A for type I ('light') gradient profile**

Passenger load coeff. / Coordinate error, m	0	0,25	**0,5**	0,75	1
25	5,9%	5,5%	5,4%	6,2%	8,0%
20	5,2%	4,8%	4,6%	5,5%	7,3%
10	3,3%	2,9%	2,6%	3,4%	5,3%
0	0,4%	0,1%	**0,0%**	0,9%	2,9%
-10	3,5%	3,3%	3,3%	3,7%	5,7%
-20	4,7%	4,6%	4,6%	5,0%	6,8%
-25	5,6%	5,6%	5,6%	5,9%	7,6%

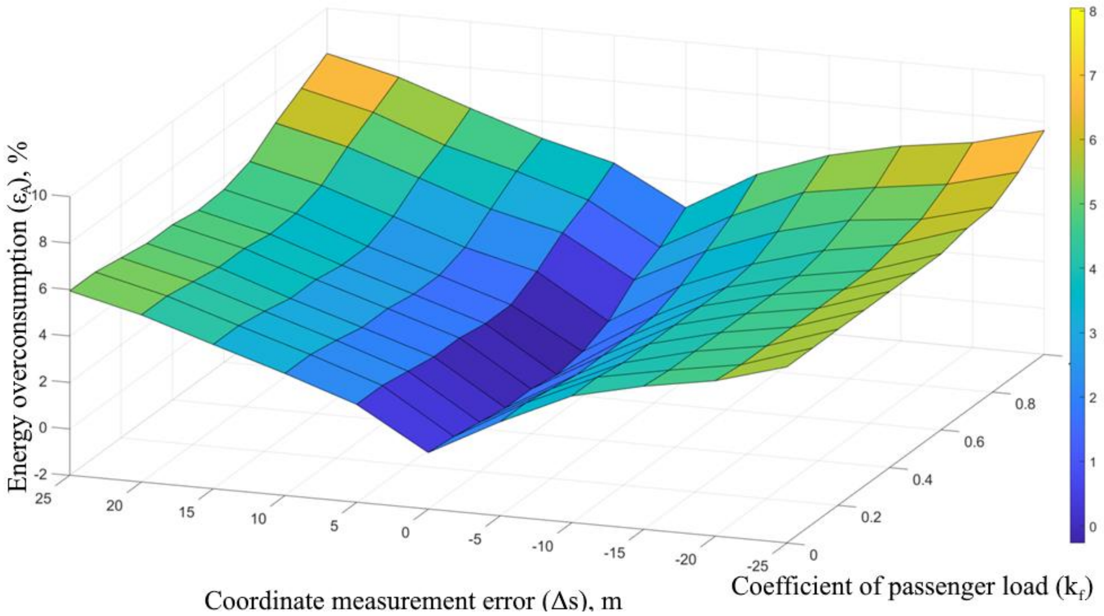

Fig. 8. **Dependency of energy overconsumption ε_A on error of measurement of train coordinate and train weight deviations for type I ('light') gradient profile**

Table 6. **Energy overconsumption ε_A for type II ('medium) gradient profile**

Passenger load coeff. / Coordinate error, m	0	0,25	**0,5**	0,75	1
25	9,9%	10,1%	10,8%	13,7%	15,5%
20	8,3%	8,4%	9,2%	12,3%	14,2%
10	5,1%	5,2%	5,7%	9,7%	11,6%
0	0,2%	0,1%	**0,0%**	5,6%	8,2%
-10	3,7%	4,0%	4,4%	8,1%	10,4%
-20	5,7%	6,0%	6,6%	9,6%	11,6%
-25	6,8%	7,2%	7,8%	10,4%	12,4%

Table 7. **Energy overconsumption ε_A for type III ('heavy') gradient profile**

Passenger load coeff. / Coordinate error, m	0	0,25	**0,5**	0,75	1
25	9,6%	13,1%	18,7%	32,0%	35,4%
20	7,3%	10,6%	15,4%	29,8%	33,6%
10	2,4%	5,1%	8,4%	24,5%	29,4%
0	-3,2%	-0,7%	**0,0%**	18,6%	24,4%
-10	2,7%	4,7%	7,3%	22,4%	27,5%
-20	7,2%	9,4%	12,7%	26,0%	31,2%
-25	9,4%	11,5%	15,1%	28,1%	33,1%

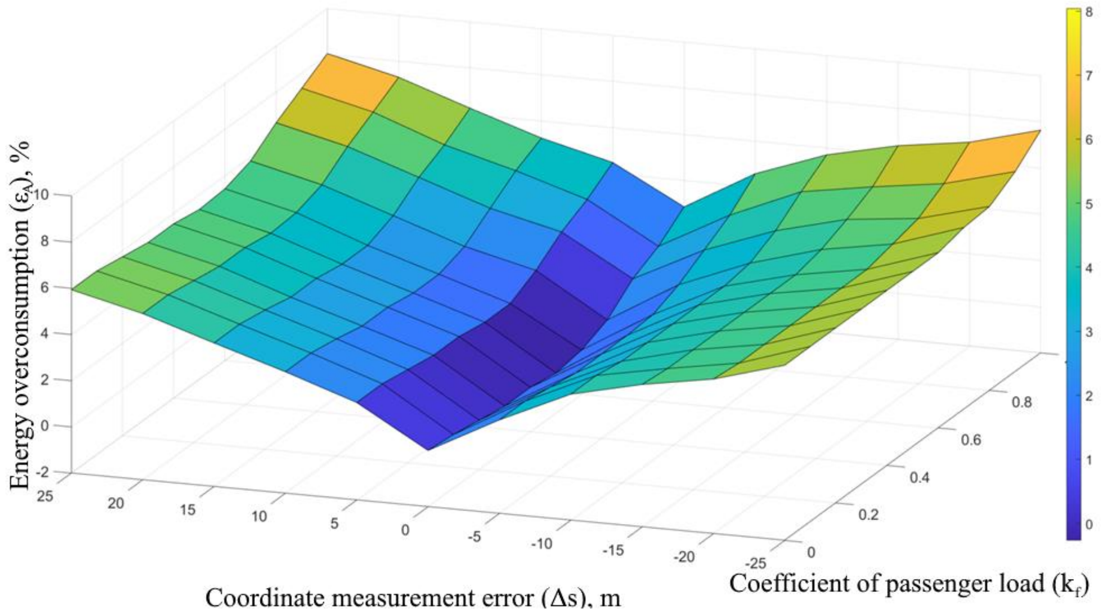

Fig. 9. **Dependency of energy overconsumption ε_A on error of measurement of train coordinate and train weight deviations for type II ('medium') gradient profile**

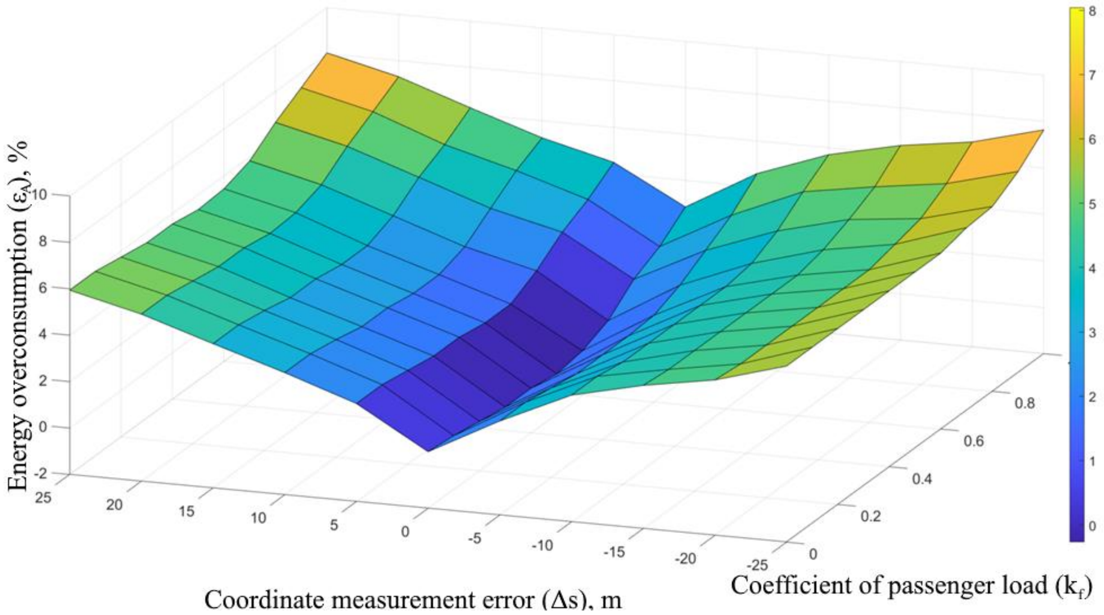

Fig. 10. **Dependency of energy overconsumption ε_A on error of measurement of train coordinate and train weight deviations for type III ('heavy) gradient profile**

Conclusions. In this study, energy consumption was analyzed for train operation along a trajectory optimized for specific conditions with varying degrees of input data reliability (based on train weight and its actual location). The analysis was performed for four typical profiles, varying in length and gradient.

Based on the analysis, the application of the rolling stock trajectory optimization model using dynamic programming by the backward induction method is substantiated. This method allows to obtain a globally optimized control law within the limits of system discreteness. The cost function of each trajectory option is determined based on the "energy-time" balance using an indefinite Lagrange multiplier, which is refined iteratively using the bisection method.

It has been shown that the speed trajectory optimization model has to be supplemented with a simulation of train dynamics, in which the automatic driving system maintains a given trajectory, with the trajectory being defined as a function of the train coordinate $V = f(s)$. The division of trajectories into "optimized" and 'actual' allows for deviations of the "actual" parameters from those for which the optimization was performed and allows for the assessment of the impact of deviations on energy consumption, i.e., the sensitivity of energy consumption to deviations from the calculated conditions.

The studies conducted made it possible to assess the degree of influence of the reliability of input information on the increase in electricity consumption and the nature of the influence depending on the profile category. As a rule, energy overconsumption increases with the complexity of the route profile (its length and gradients) – from 4% on the easiest profile to almost 40% on the "difficult" profile. On easy profiles, the greatest impact on overruns is the position measurement error. On heavier profiles, on the contrary, the greatest impact is exerted by the train's passenger load factor.

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Оцінка впливу неточностей первинної інформації на енергоспоживання рухомого складу

Анотація. *Енергоефективність систем міського залізничного транспорту є критично важливим показником, оскільки споживання енергії на тягу зазвичай становить 40-60% від загального енергоспоживання транспортної системи. У цьому дослідженні розглядається чутливість енергоспоживання до відхилень від номінальних умов при реалізації попередньо розрахованих оптимізованих траєкторій руху електрорухомого складу, розглядаючи рухомий склад з режимами роботи, типовими для приміських та міських перевезень. Для визначення глобально оптимальних стратегій керування, які мінімізують споживання енергії при дотриманні експлуатаційних обмежень, в дослідженні використовується динамічне програмування на основі принципу оптимальності Беллмана. Оптимізаційна модель розділяє ділянку колії на дискретні сегменти і використовує метод зворотного проходу для встановлення оптимальних законів управління, створюючи траєкторії швидкості як функції поточних координат поїзда на заданих поздовжніх профілях перегонів. Компроміс між енергією та часом* *представлений невизначеним множником Лагранжа для забезпечення дотримання графікового часу руху по перегону. Аналіз чутливості виконується шляхом моделювання неточностей в оцінках поточних координат поїзда та варіацій його пасажирського навантаження. Моделювання системи прицільного гальмування було реалізовано таким чином, щоб забезпечити точність зупинки у випадку неточності вимірювань. Моделювання проводилося з використанням трьох типових профілів перегонів, характерних, в першу чергу, для метрополітенів; для порівняння моделювання також проводилося на умовній ділянці з незначним постійним ухилом. Методика дослідження дозволяє кількісно оцінити ступінь перевитрат енергії, які можуть бути спричинені відхиленнями в завантаженні поїздів пасажирами та похибками в оцінці положення рухомого складу (±25 метрів), що надає інформацію для оцінки ефективності попередньо розрахованих оптимізованих траєкторій в реальних умовах експлуатації.*

Ключові слова: *оптимізація траєкторій руху, міський залізничний транспорт, енергоефективність, динамічне програмування.*

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