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**ANALYSIS OF THE TOTAL SQUARED CORRELATION FOR EVALUATING
ENSEMBLES OF UNEQUAL-ENERGY COMPLEX SIGNALS IN CODE-DIVISION
MULTIPLE-ACCESS SYSTEMS**

Candidate of Technical Sciences O. S. Zhuchenko

**АНАЛІЗ ФУНКЦІОНАЛА ЗАГАЛЬНОЇ КВАДРАТИЧНОЇ КОРЕЛЯЦІЇ ДЛЯ
ОЦІНЮВАННЯ АНСАМБЛІВ РІЗНОЕНЕРГЕТИЧНИХ СКЛАДНИХ СИГНАЛІВ
СИСТЕМ МНОЖИННОГО ДОСТУПУ З КОДОВИМ РОЗДІЛЕННЯМ**

Канд. техн. наук О. С. Жученко

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Abstract. *The objective of this study is to establish a functional relationship between the autocorrelation component of the normalized energy-weighted total squared correlation and the structural equivalent size of an unequal-energy complex signal ensemble by analyzing the autocorrelation and cross-correlation components and the dependence of the structural equivalent size on the distribution of signal energies within the ensemble. The autocorrelation and cross-correlation components of the normalized energy-weighted total squared correlation for an unequal-energy complex signal ensemble are analyzed. It is shown that the autocorrelation component is determined by the normalized sum of squared signal energies in the ensemble, whereas the cross-correlation component is expressed in terms of individual normalized multiple-access interference energies with weighting factors determined by the distribution of signal energies within the ensemble. The autocorrelation component of the normalized energy-weighted total squared correlation is shown to equal the quadratic measure of signal energy concentration, which also determines the structural equivalent size. A functional relationship is established between the autocorrelation component of the normalized energy-weighted total squared correlation and the structural equivalent size. For an equal-energy complex signal ensemble, the structural equivalent size coincides with the ensemble size, while the autocorrelation component of the normalized energy-weighted total squared correlation corresponds to a uniform distribution of signal energies. The quadratic measure of signal energy concentration is also shown to appear in the cross-correlation component of the normalized energy-weighted total squared correlation as the sum of its weighting factors. The cross-correlation component also depends on the individual normalized multiple-access interference energies, which are determined by the signal energies and their cross-correlations. It is established that the normalized energy-weighted total squared correlation differs from the structural equivalent size: a direct functional relationship with the structural equivalent size is established for the autocorrelation component, whereas the distribution of signal energies alone is insufficient to determine the cross-correlation component.*

Keywords: *unequal-energy complex signal ensemble; total squared correlation; autocorrelation component; cross-correlation component; normalized energy-weighted total squared correlation; structural equivalent size; code-division multiple-access system; multiple-access interference.*

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Анотація. Метою статті є встановлення функціонального зв'язку між автокореляційною складовою енергетично-зваженого нормованого функціонала загальної квадратичної кореляції і структурним еквівалентним об'ємом ансамблю різноенергетичних складних сигналів на основі аналізу автокореляційної та взаємкореляційної складових функціонала і залежності структурного еквівалентного об'єму від розподілу енергій сигналів ансамблю. Проаналізовано автокореляційну та взаємкореляційну складові енергетично-зваженого нормованого функціонала загальної квадратичної кореляції для ансамблю різноенергетичних складних сигналів. Показано, що автокореляційна складова визначена нормованою сумою квадратів енергій сигналів ансамблю, а взаємкореляційна складова виражена через індивідуальні нормовані енергії завад множинного доступу з ваговими множниками, які визначають розподіл енергій сигналів ансамблю. Установлено, що автокореляційна складова енергетично-зваженого нормованого функціонала загальної квадратичної кореляції дорівнює квадратичній мірі концентрації енергій сигналів ансамблю, через яку також визначено структурний еквівалентний об'єм. Між автокореляційною складовою енергетично-зваженого нормованого функціонала загальної квадратичної кореляції і структурним еквівалентним об'ємом встановлено функціональний зв'язок. Для ансамблю рівноенергетичних складних сигналів структурний еквівалентний об'єм збігається з об'ємом ансамблю, а автокореляційна складова енергетично-зваженого нормованого функціонала загальної квадратичної кореляції відповідає рівномірному розподілу енергій сигналів. Показано, що квадратична міра концентрації енергій сигналів входить також у взаємкореляційну складову енергетично-зваженого нормованого функціонала загальної квадратичної кореляції як сумарна величина її вагових множників. Значення взаємкореляційної складової також залежить від індивідуальних нормованих енергій завад множинного доступу, які визначені енергіями сигналів і їхніми взаємними кореляціями. Установлено, що енергетично-зважений нормований функціонал загальної квадратичної кореляції не є тотожним структурному еквівалентному об'єму: функціональний зв'язок зі структурним еквівалентним об'ємом безпосередньо встановлено для автокореляційної складової, тоді як для визначення взаємкореляційної складової одного розподілу енергій сигналів недостатньо.

Ключові слова: ансамбль різноенергетичних складних сигналів, функціонал загальної квадратичної кореляції, автокореляційна складова, енергетично-зважений нормований функціонал загальної квадратичної кореляції, квадратична міра концентрації енергій сигналів ансамблю, структурний еквівалентний об'єм, система множинного доступу з кодовим розділенням.

Introduction. In remote condition-monitoring systems for railway transport, wireless transmission of operational data must support long-term autonomous operation of remote devices and simultaneous transmission of critical and non-critical messages. Meeting these requirements calls for a unified wireless ultra-wideband access platform with software-controlled parameter adjustment according to current tasks, access priorities, and the number of simultaneously supported channels. At the complex-signal-ensemble level, this requirement calls for code-division multiple access. A promising direction for the

development of ultra-wideband code-division multiple-access systems is the use of complex impulse signals. In [1], ensembles of periodic pulse sequences with minimum energy interaction (hereinafter, complex time-division signal ensembles with a periodic structure) were proposed, for which no more than one pulse overlap occurs at arbitrary time shifts. The different numbers of pulses in these signals result in unequal energies of individual signals in the ensemble [2–6]; therefore, complex time-division signal ensembles with a periodic structure are unequal-energy ensembles.

Relevance of the research topic. One possible approach to evaluating the efficiency of complex signal ensembles is the use of Total Squared Correlation (TSC) [7–9]. For ensembles with unequal signal powers and/or energies, weighted variants of total squared correlation are used, in which squared correlations are assigned the corresponding weighting factors [10–14].

Another approach to evaluating unequal-energy complex signal ensembles is based on an equivalent transformation of such an ensemble into an equal-energy complex signal ensemble while preserving the total signal energy [15, 16]. The result of the equivalent transformation is the structural equivalent size, which is determined from the quadratic measure of signal energy concentration normalized to its value for an equal-energy ensemble and represents the calculated number of equal-energy signals. Numerically, it equals an approximate estimate of the normalized total energy of interfering signals and reflects the increase in interfering-signal energy for users with the lowest signal energies due to the dominance of signals from users with higher energies [15, 16]. The structural equivalent size was subsequently used in the formulation and study of performance indicators for unequal-energy complex signal ensembles and code-division multiple-access systems [17–20]. Thus, the available approaches to evaluating unequal-energy complex signal ensembles characterize the ensembles through different quantitative relationships. Therefore, establishing the relationship between total squared correlation and the structural equivalent size is a relevant task.

Analysis of recent research and publications. Total squared correlation is defined as the sum of squared correlations over all pairs of signals in the ensemble [7]. Its autocorrelation and cross-correlation components can be separated [8, 9].

For ensembles with unequal signal powers and/or energies, weighted and normalized weighted variants of total squared correlation are used; squared correlations are

assigned weighting factors that depend on signal powers or energies, so the power or energy distribution affects the weighting factors [10–14].

The distribution of signal energies can also be accounted for by an equivalent transformation of an unequal-energy complex signal ensemble into an equal-energy complex signal ensemble while preserving the total signal energy. In this transformation, the structural equivalent size is determined from the quadratic measure of signal energy concentration normalized to its value for an equal-energy ensemble and therefore depends on the distribution of signal energies [15, 16].

At the same time, the normalized energy-weighted total squared correlation and the structural equivalent size are obtained within different approaches to evaluating unequal-energy complex signal ensembles. The relationship between the components of the normalized energy-weighted total squared correlation and the structural equivalent size has yet to be established.

Defining the purpose and objectives of the research. The objective of this article is to establish a functional relationship between the autocorrelation component of the normalized energy-weighted total squared correlation and the structural equivalent size of an unequal-energy complex signal ensemble by analyzing the autocorrelation and cross-correlation components and the dependence of the structural equivalent size on the distribution of signal energies in the ensemble.

The main part of the research. Consider total squared correlation (TSC), defined as the sum of squared correlations over all pairs of signals in the ensemble [7]. For an ensemble of size L , TSC is given by:

$$\text{TSC} = \sum_{i=1}^L \sum_{j=1}^L \left(R_{ij}^{(max)} \right)^2,$$

where L – ensemble size;

$R_{ij}^{(max)}$ – maximum correlation value between signals i and j , $i, j = 1, \dots, L$.

For ensembles with unequal signal energies, weighted and normalized weighted variants of total squared correlation are used [10–14]. For the subsequent analysis, we use the normalized energy-weighted total squared correlation, denoted by TSC_{enwt} . Let the energy of signal i be denoted by E_i , and define the total signal energy of the ensemble E_Σ as

$$E_\Sigma = \sum_{i=1}^L E_i.$$

Accounting for the signal energies E_i , the normalized energy-weighted total squared correlation TSC_{enwt} can be written as

$$TSC_{\text{enwt}} = \frac{1}{E_\Sigma^2} \sum_{i=1}^L \sum_{j=1}^L E_i E_j \left(R_{ij}^{(\max)}\right)^2,$$

where the squared correlation $\left(R_{ij}^{(\max)}\right)^2$ is assigned the weighting factor $E_i E_j / E_\Sigma^2$. The quantity TSC_{enwt} is dimensionless because of normalization by E_Σ^2 .

For the subsequent analysis, decompose the normalized energy-weighted total squared correlation into components according to whether the signal indices coincide [8, 9]. Terms with $i = j$ define the autocorrelation component, whereas terms with $i \neq j$ define the cross-correlation component:

$$TSC_{\text{enwt}} = TSC_{\text{enwt}}^{(i=j)} + TSC_{\text{enwt}}^{(i \neq j)},$$

where $TSC_{\text{enwt}}^{(i=j)}$ – autocorrelation component of the normalized energy-weighted total squared correlation;

$TSC_{\text{enwt}}^{(i \neq j)}$ – cross-correlation component of the normalized energy-weighted total squared correlation.

For $\left(R_{ii}^{(\max)}\right)^2 = 1$, the autocorrelation component of the normalized energy-weighted total squared correlation $TSC_{\text{enwt}}^{(i=j)}$ is given by

$$TSC_{\text{enwt}}^{(i=j)} = \frac{1}{E_\Sigma^2} \sum_{i=1}^L E_i^2,$$

which accounts for the distribution of signal energies E_i in the ensemble. Cross-correlations between signals do not enter this relationship.

The cross-correlation component of the normalized energy-weighted total squared correlation $TSC_{\text{enwt}}^{(i \neq j)}$ is given by:

$$TSC_{\text{enwt}}^{(i \neq j)} = \frac{1}{E_\Sigma^2} \sum_{i=1}^L \sum_{j=1, j \neq i}^L E_i E_j \left(R_{ij}^{(\max)}\right)^2.$$

Next, consider the structural equivalent size $L_{\text{eq}}^{(+)}$, defined as the calculated number of equal-energy complex signals in an ensemble equivalent to the original unequal-energy complex signal ensemble in terms of total signal energy [15, 16]. The structural equivalent size is obtained by multiplying the ensemble size L by a factor formed by normalizing the quadratic measure of signal energy concentration to $1/L$; the concentration measure is κ_E :

$$L_{\text{eq}}^{(+)} = L \frac{\kappa_E}{1/L} = L^2 \kappa_E = L^2 \frac{\sum_{i=1}^L E_i^2}{(\sum_{i=1}^L E_i)^2} = L^2 \frac{\sum_{i=1}^L E_i^2}{E_\Sigma^2},$$

where the concentration measure κ_E is determined by the distribution of signal energies in the ensemble:

$$\kappa_E = \frac{\sum_{i=1}^L E_i^2}{(\sum_{i=1}^L E_i)^2} = \frac{\sum_{i=1}^L E_i^2}{E_\Sigma^2}.$$

For an equal-energy complex signal ensemble with $E_i = E$, the concentration measure takes the value $\kappa_E = 1/L$.

The structural equivalent size $L_{eq}^{(+)}$ is numerically equal to an approximate estimate of the normalized total energy of interfering signals in the equivalent equal-energy complex signal ensemble when the approximation $L_{eq}^{(+)} - 1 \approx L_{eq}^{(+)}$ is used under the condition that $L_{eq}^{(+)}$ is sufficiently large, whereas without this approximation the normalized total energy of interfering signals is $L_{eq}^{(+)} - 1$.

Now use the concentration measure κ_E to express the relationship obtained above for the autocorrelation component $TSC_{enwt}^{(i=j)}$ of the normalized energy-weighted total squared correlation:

$$TSC_{enwt}^{(i=j)} = \frac{1}{E_{\Sigma}^2} \sum_{i=1}^L E_i^2 = \kappa_E.$$

Thus, the structural equivalent size $L_{eq}^{(+)}$ and the autocorrelation component $TSC_{enwt}^{(i=j)}$ of the normalized energy-weighted total squared correlation are both expressed through the quadratic measure of signal energy concentration κ_E . Substituting $\kappa_E = L_{eq}^{(+)}/L^2$ gives the functional relationship between the autocorrelation component $TSC_{enwt}^{(i=j)}$ and the structural equivalent size $L_{eq}^{(+)}$:

$$TSC_{enwt}^{(i=j)} = \kappa_E = \frac{L_{eq}^{(+)}}{L^2}.$$

Thus, the autocorrelation component $TSC_{enwt}^{(i=j)}$ of the normalized energy-weighted total squared correlation TSC_{enwt} is determined by the distribution of signal energies in the ensemble and equals the quadratic measure of signal energy concentration. The structural equivalent size is also determined through the concentration measure κ_E , normalized to its value for an equal-energy ensemble; consequently, a functional relationship is established between the autocorrelation component and the structural equivalent size.

Next, consider the cross-correlation component $TSC_{enwt}^{(i \neq j)}$ of the normalized energy-weighted total squared correlation TSC_{enwt} . For user i , define the individual multiple-access interference energy $E_{I,i}$ as the sum of the energies of interfering users' signals weighted by the squares of the corresponding cross-correlations:

$$E_{I,i} = \sum_{j=1, j \neq i}^L E_j \left(R_{ij}^{(max)} \right)^2.$$

Substituting the definition of $E_{I,i}$ into the expression for the cross-correlation component gives:

$$\begin{aligned} TSC_{enwt}^{(i \neq j)} &= \frac{1}{E_{\Sigma}^2} \sum_{i=1}^L \sum_{j=1, j \neq i}^L E_i E_j \left(R_{ij}^{(max)} \right)^2 = \\ &= \frac{1}{E_{\Sigma}^2} \sum_{i=1}^L E_i \left(\sum_{j=1, j \neq i}^L E_j \left(R_{ij}^{(max)} \right)^2 \right) = \frac{1}{E_{\Sigma}^2} \sum_{i=1}^L E_i E_{I,i}. \end{aligned}$$

In this relationship, the individual multiple-access interference energies $E_{I,i}$ enter the sum with weights determined by the energies of the corresponding signals. For the

subsequent transformation, introduce the weighting factors w_i :

$$w_i = \frac{E_i}{E_\Sigma}, \quad \sum_{i=1}^L w_i = 1.$$

The cross-correlation component $TSC_{\text{enwt}}^{(i \neq j)}$ is then given by:

$$TSC_{\text{enwt}}^{(i \neq j)} = \sum_{i=1}^L w_i \frac{E_{I,i}}{E_\Sigma},$$

where, for each user i , the ratio $E_{I,i}/E_i$ is the individual normalized multiple-access interference energy. Since $E_i/E_\Sigma = w_i$, the last expression can be transformed to:

$$TSC_{\text{enwt}}^{(i \neq j)} = \sum_{i=1}^L \frac{E_i^2}{E_\Sigma^2} \frac{E_{I,i}}{E_i}.$$

Thus, the cross-correlation component $TSC_{\text{enwt}}^{(i \neq j)}$ is expressed in terms of the individual normalized multiple-access interference energies $E_{I,i}/E_i$ with weighting factors E_i^2/E_Σ^2 . The sum of the weighting factors equals the quadratic measure of signal energy concentration in the ensemble κ_E :

$$\sum_{i=1}^L \frac{E_i^2}{E_\Sigma^2} = \kappa_E,$$

where the concentration measure κ_E is a common factor of the cross-correlation component $TSC_{\text{enwt}}^{(i \neq j)}$, while the quantities $E_{I,i}/E_i$ depend on the energies of the interfering signals and the squares of their cross-correlations with signal i . Thus, the cross-correlation component $TSC_{\text{enwt}}^{(i \neq j)}$ simultaneously

which shows that the quadratic measure of signal energy concentration κ_E enters the relationship for the cross-correlation component $TSC_{\text{enwt}}^{(i \neq j)}$, where κ_E gives the sum of the weighting factors applied to the individual normalized multiple-access interference energies $E_{I,i}/E_i$. Write the weighting factor as:

$$\frac{E_i^2}{E_\Sigma^2} = \kappa_E \frac{E_i^2}{\sum_{j=1}^L E_j^2},$$

where the fractions $E_i^2/\sum_{j=1}^L E_j^2$ specify the fraction of the sum of squared signal energies attributable to signal i , and the sum of these fractions equals one:

$$\sum_{i=1}^L \frac{E_i^2}{\sum_{j=1}^L E_j^2} = 1.$$

Thus, the weighting factor E_i^2/E_Σ^2 can be written as the product of two components: the concentration measure κ_E and the fraction $E_i^2/\sum_{j=1}^L E_j^2$. The quadratic measure of signal energy concentration κ_E determines the overall quadratic concentration of energy in the ensemble, whereas the ratio $E_i^2/\sum_{j=1}^L E_j^2$ gives the fraction of the sum of squared signal energies associated with signal i .

Substituting $E_i^2/E_\Sigma^2 = \kappa_E E_i^2/\sum_{j=1}^L E_j^2$ into the expression for $TSC_{\text{enwt}}^{(i \neq j)}$ gives:

$$TSC_{\text{enwt}}^{(i \neq j)} = \sum_{i=1}^L \frac{E_i^2}{E_\Sigma^2} \frac{E_{I,i}}{E_i} = \kappa_E \sum_{i=1}^L \frac{E_i^2}{\sum_{j=1}^L E_j^2} \frac{E_{I,i}}{E_i},$$

accounts for the distribution of signal energies in the ensemble and their cross-correlations.

Write the normalized energy-weighted total squared correlation TSC_{enwt} as the sum of the autocorrelation $TSC_{\text{enwt}}^{(i=j)}$ and cross-correlation $TSC_{\text{enwt}}^{(i \neq j)}$ components:

$$\begin{aligned} \text{TSC}_{\text{enwt}} &= \text{TSC}_{\text{enwt}}^{(i=j)} + \text{TSC}_{\text{enwt}}^{(i \neq j)} = \\ &= \kappa_E + \kappa_E \sum_{i=1}^L \frac{E_i^2}{\sum_{j=1}^L E_j^2} \frac{E_{I,i}}{E_i} = \kappa_E \left(1 + \sum_{i=1}^L \frac{E_i^2}{\sum_{j=1}^L E_j^2} \frac{E_{I,i}}{E_i} \right). \end{aligned}$$

The resulting relationship shows two factors that determine the normalized energy-weighted total squared correlation TSC_{enwt} . The quadratic measure of signal energy concentration κ_E characterizes the distribution of signal energies in the ensemble and is a common factor of both the autocorrelation and cross-correlation components. The expression in parentheses accounts for the individual normalized multiple-access interference energies $E_{I,i}/E_i$, which also depend on the cross-correlations between signals. At the same value of κ_E , TSC_{enwt} can take different values if the quantities $E_{I,i}/E_i$ differ between signal ensembles.

Since the structural equivalent size $L_{\text{eq}}^{(+)}$ satisfies the relationship $L_{\text{eq}}^{(+)} = L^2 \kappa_E$, the normalized energy-weighted total squared correlation TSC_{enwt} can also be expressed in terms of the structural equivalent size:

$$\text{TSC}_{\text{enwt}} = \frac{L_{\text{eq}}^{(+)}}{L^2} \left(1 + \sum_{i=1}^L \frac{E_i^2}{\sum_{j=1}^L E_j^2} \frac{E_{I,i}}{E_i} \right),$$

Thus, for an equal-energy complex signal ensemble, the cross-correlation component $\text{TSC}_{\text{enwt}}^{(i \neq j)}$ of the normalized energy-weighted total squared correlation TSC_{enwt} is determined by the sum of squared cross-correlations with weighting factor $1/L^2$. The structural equivalent size $L_{\text{eq}}^{(+)}$ for such an ensemble coincides with the ensemble size L .

where $L_{\text{eq}}^{(+)}/L^2 = \kappa_E$ directly determines the autocorrelation component $\text{TSC}_{\text{enwt}}^{(i=j)}$ and is a common factor of the cross-correlation component $\text{TSC}_{\text{enwt}}^{(i \neq j)}$.

For an equal-energy complex signal ensemble, where $E_i = E$, $i = 1, \dots, L$, we obtain:

$$\kappa_E = \frac{\sum_{i=1}^L E_i^2}{(LE)^2} = \frac{LE^2}{L^2 E^2} = \frac{1}{L},$$

and for each signal, the fraction of the sum of squared signal energies associated with signal i is:

$$\frac{E_i^2}{\sum_{j=1}^L E_j^2} = \frac{E^2}{LE^2} = \frac{1}{L}.$$

Substituting $\kappa_E = 1/L$, $E_i^2 / \sum_{j=1}^L E_j^2 = 1/L$, $E_{I,i} = E \sum_{j=1, j \neq i}^L (R_{ij}^{(\max)})^2$, and $E_{I,i}/E = \sum_{j=1, j \neq i}^L (R_{ij}^{(\max)})^2$ into the expression for the cross-correlation component $\text{TSC}_{\text{enwt}}^{(i \neq j)}$ gives:

$$\text{TSC}_{\text{enwt}}^{(i \neq j)} = \kappa_E \sum_{i=1}^L \frac{E_i^2}{\sum_{j=1}^L E_j^2} \frac{E_{I,i}}{E_i} = \frac{1}{L} \sum_{i=1}^L \frac{1}{L} \frac{E_{I,i}}{E} = \frac{1}{L^2} \sum_{i=1}^L \frac{E_{I,i}}{E} = \frac{1}{L^2} \sum_{i=1}^L \sum_{j=1, j \neq i}^L (R_{ij}^{(\max)})^2.$$

Conclusions. The autocorrelation and cross-correlation components of the normalized energy-weighted total squared correlation for an unequal-energy complex signal ensemble are analyzed. It is shown that the autocorrelation component is determined by the normalized sum of squared signal energies in the ensemble, whereas the cross-correlation

component is expressed in terms of individual normalized multiple-access interference energies with weighting factors determined by the distribution of signal energies within the ensemble. The autocorrelation component of the normalized energy-weighted total squared correlation is shown to equal the quadratic measure of signal energy concentration, which also determines the structural equivalent size. A functional relationship is established between the autocorrelation component of the normalized energy-weighted total squared correlation and the structural equivalent size. For an equal-energy complex signal ensemble, the structural equivalent size coincides with the ensemble size, while the autocorrelation component of the normalized energy-weighted total squared correlation corresponds to a

uniform distribution of signal energies. The quadratic measure of signal energy concentration is also shown to appear in the cross-correlation component of the normalized energy-weighted total squared correlation as the sum of its weighting factors. The cross-correlation component also depends on the individual normalized multiple-access interference energies, which are determined by the signal energies and their cross-correlations. It is established that the normalized energy-weighted total squared correlation differs from the structural equivalent size: a direct functional relationship with the structural equivalent size is established for the autocorrelation component, whereas the distribution of signal energies alone is insufficient to determine the cross-correlation component.

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Zhuchenko Oleksandr Serhiiovych, Candidate of Technical Sciences, Associate Professor, Associate Professor of Department of Transport Communication, Ukrainian State University of Railway Transport, Kharkiv, Ukraine. Tel.: +38 (063) 779-44-12. E-mail: n030201@gmail.com. ORCID ID: 0000-0003-3275-810X.

Жученко Олександр Сергійович, кандидат технічних наук, доцент, доцент кафедри транспортного зв'язку, Український державний університет залізничного транспорту, Харків, Україна. Тел.: +38 (063) 779-44-12. E-mail: n030201@gmail.com. ORCID ID: 0000-0003-3275-810X.

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